Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2016

Solutions to Assignment #1
Random process?

Consider the following two sequences of one hundred heads and tails:

Sequence #1:

HHTHTTTHTHHTTTTHTTHTHTTHHTTHTHHTTHTTTHTHHHHTTTHTTH
HTTTTHTHHHTHTTHTTHHTHTHHTTHHHHTHHHTTTHTTTHTHHHTTHT

Sequence #2:

HHTHTHTHTHHTHHHTHHHTHHTHHHTTHHTHHTTTHHTHTTTTTTHTTT
THTTTTTTTHHTHTTHHTTTTTHHHTHTTHTTHHTTTHHHTTHHHHTTTT

One of these sequences was generated by actually spinning a toonie one hundred times; the other was generated by your instructor sitting at his computer and hitting the "H" and "T" keys one hundred times between them and trying to make it seem random.

1. Try to figure out which sequence was generated by spinning a coin and which was not. Give your reasoning! *[10]*

SOLUTION. There are a number of things one could look for that could help distinguish a human-generated sequence from a an actual coin-generated sequence. None is completely reliable – after all, any particular sequence can arise randomly. Here are three fairly simple criteria, out of many possible:

i. Number of Hs and number of Ts. Humans trying to make a random sequence tend to try to keep the number of Hs and Ts approximately even, but a real coin doesn't care what happenned on previous flips. Unfortunately, this criterion isn't very useful, especially for longer sequences, because it is highly probable that a long sequence of flips of a fair coin will have an approximately equal number of Hs as Ts. In our case, sequence #1 has 47 Hs and 53 Ts, while sequence #2 has 46 Hs and 54 Ts, so this criterion is basically completely inconclusive for the given sequences.

ii. Long runs of Hs or Ts. Due to the tendency to weigh recent events more heavily and the fact that they try to keep the number of Hs and Ts approximately even, a human-generated sequence is very likely to have fewer longish runs of all Hs or all Ts than a truly random sequence is likely to. In our case, sequence #1 has no run longer than four, while sequence #2 has a run of seven and a couple of runs of five. This suggests that sequence #2 is probably the coin-generated one and sequence #1 was made by human hands.

iii. Number of runs or number of transitions between Hs and Ts. Since humans tend not to have longer runs and tend to try to even out the number of Hs and Ts in the short run, a human-generated sequence will tend to have more (but shorter) runs of consecutive Hs or consecutive Ts than a randomly generated one. (Equivalently, a human-generated sequence will probably transition from H to T or *vice versa* more frequently than a truly random one does.) In our case, sequence #1 has 57 runs (55 transitions) and sequence #2 has 49 runs (47 transitions), suggesting again that sequence #2 is probably the coin-generated one and sequence #1 is probably the human-generated one.

So the way to bet is that sequence #2 is the genuinely coin-generated one, and sequence #1 is not. [This is indeed the case!] ■