Mathematics 1550H – Introduction to probability
Trent University, Summer 2016
Quizzes

Quiz #1. Wednesday, 22 June, 2016. *[10 minutes]*

A fair coin is tossed. If the coin comes up heads, a fair three-sided die with faces labelled 1, 2, and 3, respectively, is rolled once; if the coin comes up tails, a fair three-sided die with faces labelled 2, 3, and 4, respectively, is rolled once.

1. What are the sample space and the probability mass function for this experiment? *[2]*
2. Draw the complete tree diagram for this experiment. *[1.5]*
3. What is the probability that the die roll in the second stage of the experiment produced a 2? *[1.5]*

Quiz #2. Monday, 27 June, 2016. *[10 minutes]*

Consider the function $f(t) = \begin{cases} t & 0 \le t \le 1 \\ 2 - t & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$.

0. Sketch the graph of $f(t)$. *[1]*
1. Verify that $f(t)$ is a probability density function. *[2]*
2. Compute $P\left(\left[\frac{1}{2}, \infty\right)\right)$. *[2]*

Quiz #3. Wednesday, 29 June, 2016. *[10 minutes]*

A five-card hand is randomly drawn, all at once and without replacement, from a standard 52-card deck.

1. What is the probability that the hand is a *full house*, consisting of three of one kind and two of another kind? *[5]*

Quiz #4. Monday, 4 July, 2016. *[10 minutes]*

1. The four cards $A\heartsuit$, $A\diamondsuit$, $A\clubsuit$, and $A\spadesuit$ are laid out for you in that order. You shuffle the four cards and lay them out again in a random order. What is the probability that none of the four cards ends up in the same position it was in when the hand was originally given to you? *[5]*

Quiz #5. Wednesday, 6 July, 2016. *[10 minutes]*

A fair coin is tossed four times. Let A be the event that there were at least three heads, and let B be the event that at least one of the first two tosses was a head.

1. Determine whether A and B are independent or not. *[3]*
2. Compute $P(A|B)$. *[2]*

Quiz #6. Wednesday, 13 July, 2016. *[10 minutes]*

An otherwise fair seven-sided die has single faces labelled 1, 2, 4, and 5, respectively, and three faces labelled 3. The die is rolled once; let X be the number that comes up on the roll.

1. Compute the expected value, $E(X)$, of X. *[2.5]*
2. Compute the variance, $V(X)$, of X. *[2.5]*

Quiz #7. Monday, 18 July, 2016. *[10 minutes]*

Suppose that the continuous random variable X has the probability density function

$$f(t) = \begin{cases} \frac{3}{4}\left(1 - t^2\right) & -1 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

1. Compute the expected value, $E(X)$, of X. *[2.5]*
2. Compute the variance, $V(X)$, of X. *[2.5]*

Quiz #8. Wednesday, 20 July, 2016. *[10 minutes]*

Suppose the continuous random variable X has a normal distribution with expected value $\mu = 10$ and standard deviation $\sigma = 5$.

1. Compute $P(7 \le X \le 17)$. *[3]*
2. Find the value of c for which $P(X \le c) \approx 0.7734$. *[2]*

Quiz #9. Monday, 25 July, 2016. *[however many minutes]*

Suppose the discrete random variables X and Y are jointly distributed according to the following table:

$X \backslash Y$	-1	2
2	0.1	0.1
3	0.2	0.1
5	0.3	0.2

1. Compute the expected values $E(X)$ and $E(Y)$, variances $V(X)$ and $V(Y)$, and covariance $\text{Cov}(X, Y)$ of X and Y. *[5]*

Quiz #10. Wednesday, 27 July, 2016. *[10 minutes]*

Suppose X is a random variable with $X \ge 0$, expected value $E(X) = 10$, and variance $V(X) = 4$.

1. What can you say about $P(X < 20)$ with the help of Markov's Inequality? *[2]*
2. What can you say about $P(X < 20)$ with the help of Chebyshev's Inequality? *[3]*