Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2016

Assignment #5
Chebyshev's (In)equality
Due on Monday, 25 July, 2016.

A common version of *Chebyshev's Inequality* states that a random variable X with expected value $\mu = E(X)$ and standard deviation $\sigma = \sqrt{V(X)}$ must satisfy

$$P\left(|X-\mu| \geq k\sigma\right) \leq \frac{1}{k^2}$$

for any real number $k > 0$.

The great advantage of this inequality is that it can be used without knowing anything about the probability distribution of the random variable X, except, of course, its mean and standard deviation. (If the mean and/or the standard deviation are not defined or are unknown for X, Chebyshev's Inequality isn't of much use.) Note too that it is of interest only if $k > 1$, since if $k \leq 1$, then $\frac{1}{k^2} \geq 1$ and any probability must be ≤ 1. There are many variations on Chebyshev's Inequality; the one given above is one of the two most common forms, the other of which we'll see in class.

1. Find a probability density function $f(x)$ for a continuous random variable such that a continuous random variable X with this density will have $\mu = 0$, $\sigma = 1$, and satisfy $P\left(|X| \geq 2\right) = \frac{1}{4}$. *[5]*

2. Given a fixed real number $k > 1$, any fixed real number μ, and any fixed real number $\sigma > 0$, find a probability density function $g(x)$ such that a continuous random variable X with this density will have mean μ, standard deviation σ, and satisfy $P\left(|X-\mu| \geq k\sigma\right) = \frac{1}{k^2}$. *[5]*

NOTE: The point of **2**, in particular, is that Chebyshev's Inequality can't really be improved upon unless you have additional information about the distribution of the random variable X.