Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2015

Distributions
The Short Form

Discrete

1. *Discrete Uniform.* n equally likely outcomes for some $n \geq 1$.
Probability function: $p(x) = P(X = x) = \dfrac{1}{n}$.

2. *Bernoulli Trial.* Two outcomes with probability of success p and of failure $q = 1 - p$.
Probability function: $p(x) = P(\text{success}) = p$.
Expected value: $\mu = E(X) = p$ *Variance:* $\sigma^2 = V(X) = pq$

3. *Binomial.* n Bernoulli trials, with probability of success p and of failure $q = 1 - p$.
Probability function: $p(x) = P(x \text{ successes}) = \dbinom{n}{x} p^x q^{n-x}$, where $0 \leq x \leq n$.
Expected value: $\mu = E(X) = np$ *Variance:* $\sigma^2 = V(X) = npq$

4. *Geometric.* Bernoulli trials repeated until the first success, with probability of success p and of failure $q = 1 - p$.
Probability function: $p(x) = P(\text{first success on } x\text{th trial}) = q^{x-1}p$
Expected value: $\mu = E(X) = \dfrac{1}{p}$ *Variance:* $\sigma^2 = V(X) = \dfrac{q}{p^2}$

5. *Negative Binomial.* Bernoulli trials repeated until the kth success, with probability of success p and of failure $q = 1 - p$.
Probability function: $p(x) = P(k \text{ success on } x\text{th trial}) = \dbinom{x-1}{k-1} p^k q^{x-k}$
Expected value: $\mu = E(X) = \dfrac{k}{p}$ *Variance:* $\sigma^2 = V(X) = \dfrac{kq}{p^2}$

6. *Poisson.* Approximates the Binomial distribution when n is large.
Probability function: $p(x) = P() = \dfrac{e^{-\lambda}\lambda^x}{x!}$
Expected value: $\mu = E(X) = \lambda$ *Variance:* $\sigma^2 = V(X) = \lambda$

Continuous

7. *Continuous Uniform.*
Density function: $f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$
Expected value: $\mu = E(X) = \dfrac{a+b}{2}$ *Variance:* $\sigma^2 = V(X) = \dfrac{(b-a)^2}{12}$

8. *Exponential.*
Density function: $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$
Expected value: $\mu = E(X) = \dfrac{1}{\lambda}$ *Variance:* $\sigma^2 = V(X) = \dfrac{1}{\lambda^2}$

over

9. *Standard normal.*
Density function: $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$
Expected value: $\mu = E(X) = 0$ *Variance:* $\sigma^2 = V(X) = 1$

10. *Normal.* ... with mean μ and standard deviation σ.
Density function: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
Expected value: $E(X) = \mu$ *Variance:* $V(X) = \sigma^2$