Trent University, Summer 2015

MATH 1550H Test

Monday, 11 July, 2015

Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do any *three (3)* of **a**–**e**. *[12 = 3 × 4 each]*

a. A fair trihedral (3-sided) die (with faces numbered 1, 2, and 3, respectively) is rolled twice. Let X be the sum of the two rolls. Compute the probability that X is even.

b. How many 3-letter sequences can be formed using the letters in the word "pastrami" if the two "a"s cannot be distinguished?

c. Five cards are simultaneously drawn at random from a standard 52-card deck. What is the probability of drawing a full house (3 of a kind plus 2 of a kind)?

d. A fair coin is tossed three times, and let X be the number of heads minus the number of tails. Find the probability function of X.

e. Suppose the continuous random variable W has a normal distribution with $\mu = 2$ and $\sigma = 3$. Compute $P(W \leq 5)$.

2. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Compute $P(A|B) + P(\bar{A}|B)$, where A and B are events in a sample space Ω.

b. A fair die is rolled twice. Let A be the event that the two rolls give a different number and B be the event that the sum of the two rolls is even. Determine whether A and B are independent or not.

c. Let $f(x) = \begin{cases} x^{-2} & x \geq 1 \\ 0 & x < 1 \end{cases}$ be the probability density function of the continuous random varable X. Let A be the event that $X \leq 2$ and let B be the event that $0 \leq X \leq 3$. Compute $P(A|B)$.

3. Do any *one (1)* of **a** or **b**. *[8 = 1 × 8 each]*

a. Suppose Z is a continuous random variable with an exponential distribution, so it has density function $h(z) = \begin{cases} \lambda e^{-\lambda z} & z \geq 0 \\ 0 & z < 0 \end{cases}$ for some $\lambda > 0$, and suppose $P(Z \leq 2) = \frac{1}{2}$. Determine λ.

b. A bin contains five blue and five red balls. If balls are drawn randomly, without replacement, from the bin until a second ball of the same colour as the first one drawn appears, what is probability that a total of at most five balls will be drawn?

[Total = 30]

Bonus: For each $n \geq 1$, describe a shape that would give you a fair n-sided die. *[1]*