Mathematics 1550H – Introduction to Probability
TRENT UNIVERSITY, Summer 2015

Quizzes

Quiz #1. Wednesday, 24 June, 2015. *[10 minutes]*

A fair (standard six-sided) die and a fair (standard two-sided) coin are tossed simultaneously. Let b and c be the outcomes of the die and coin tosses respectively. Since the die and coin are fair, any pair (b, c) is as likely to turn up as any other.

1. What is the sample space Ω? How many possible outcomes are there in Ω? *[2]*
2. Let A be the event that b is odd and $c = T$. Find $P(A)$. *[3]*

Quiz #2. Monday, 29 June, 2015. *[10 minutes]*

Do *one* of questions 1 and 2.

1. A fair coin is tossed until it comes up heads. What is the probability that it will be tossed at least four times? *[5]*
2. Suppose X is a continuous random variable with a uniform distribution on $[0, 2]$, so it has $f(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 2 \\ 0 & x < 0 \text{ or } x > 2 \end{cases}$ as its probability density function. Compute $P\left(X \le \frac{3}{2}\right)$. *[5]*

Quiz #3. Monday, 6 July, 2015. *[12 minutes]*

1. You are given ten books, eight of which are unique and the other two are indistinguishable from each other, though the two identical ones are different from the other eight. How many ways are there to arrange the ten books on two shelves, each of which could hold all ten books? *[5]*

Quiz #4. Wednesday, 8 July, 2015. *[15 minutes]*

A jar initially contains nine chocolate chip and seven butter cookies. While doing your probability homework, you eat five cookies, reaching into the jar and picking one at random each time. You are concentrating hard on your homework, so you don't really notice what kind you eat.

1. If the the last two of the five you eat are butter cookies, what is the probability that first three were chocolate chip cookies? *[5]*

Quiz #5. Wednesday, 15 July, 2015. *[12 minutes]*

A fair die is rolled five times. Let X be the number of times that the roll came up 3 or 6.

1. What is the probability function of X? *[2.5]*
2. Compute the expected value $E(X)$. *[2.5]*

Quiz #6. Monday, 20 July, 2015. *[15 minutes]*

1. Suppose the continuous random variable X has probability density $f(x) = \begin{cases} 3x^{-4} & x \ge 1 \\ 0 & x < 1 \end{cases}$.
   Compute the expected value $E(X)$ and variance $V(X)$ of X. *[5]*

Take-Home Quiz #7. *Due on Wednesday, 22 July, 2015. [2 days]*

Two people play poker in the following way. They spread a standard deck of 52 cards face up on the table so that they can see all the cards. Player 1 draws a hand of five cards by picking any five he or she chooses. Player 2 then does the same, picking any five of the remaining cards. Player 1 now has the option of keeping the hand originally drawn or discarding all of it and picking a new hand of five cards from the cards remaining. Player 2 may now do likewise, but may not use any cards that Player 1 discarded. The playern with the higher hand then wins. Suits have equal value, so that two flushes tie unless one is made of higher cards. (The cards are ranked A K Q J 10 9 . . . 2, from highest to lowest.)

1. What hand should Player 1 initially draw to guarantee victory? *[5]*

Quiz #8. Wednesday, 22 July, 2015. *[15 minutes]*

1. Suppose X and Y are independent discrete random variables which have the same probability function, namely $m(k) = \begin{cases} \frac{1}{8} & k = 0 \\ \frac{1}{2} & k = 1 \\ \frac{3}{8} & k = 2 \\ 0 & \text{otherwise} \end{cases}$. Find the probability function of $Z = X + Y$. *[5]*

Quiz #9. Monday, 27 July, 2015. *[15 minutes]*

1. Suppose X is a random variable with expected value $5 = E(X)$ and variance $4 = \sigma^2 = V(X)$, which is symmetric about its mean, that is, $P(X - 5 \leq -a) = P(X - 5 \geq a)$ for all $a > 0$. Use the symmetry of X and Chebyshev's Inequality to estimate $P(X \geq 13)$. *[5]*

Quiz #10. Wednesday, 29 July, 2015. *[15 minutes]*

Suppose the discrete random variables X and Y are jointly distributed, with their joint probability function $p(x, y)$ being given by the table below.

$X \backslash Y$	1	2
0	0.2	0.1
2	0.2	0.2
4	0.1	0.2

1. Compute $\text{Cov}(X, Y)$, the covariance of X and Y. *[5]*