**Mathematics 1550H – Introduction to Probability**
TRENT UNIVERSITY, Summer 2015

**Assignment #5**
*Due on Monday, 27 July, 2015.*
**Chebyshev's (In)equality**

*Chebyshev's Inequality* states that a random variable $X$ with mean $\mu = E(X)$ and standard deviation $\sigma = \sqrt{V(X)}$ must satisfy

$$P\left(|X - \mu| \geq k\sigma\right) \leq \frac{1}{k^2}$$

for any real number $k > 0$.

The great advantage of this inequality is that it can be used without knowing anything about the probability distribution of the random variable $X$, except, of course, its mean and standard deviation. (If the mean and/or the standard deviation are not defined or are unknown for $X$, Chebyshev's Inequality obviously cannot be used.) Note also that it is really only useful if $k > 1$, since if $k \leq 1$, then $\frac{1}{k^2} \geq 1$ and any probability must be $\leq 1$. There are many variations and extensions of the basic form of Chebyshev's Inequality for various specialized contexts.

**1.** Find a probability density function $f(x)$ for a continuous random variable such that a continuous random variable $X$ with this density will have $\mu = 0$, $\sigma = 1$, and satisfy $P\left(|X| \geq 2\right) = \frac{1}{4}$. *[5]*

**2.** Given a fixed real number $k > 1$, any fixed real number $\mu$, and any fixed real number $\sigma > 0$, find a probability density function $g(x)$ such that a continuous random variable $X$ with this density will have mean $\mu$, standard deviation $\sigma$, and satisfy $P\left(|X - \mu| \geq k\sigma\right) = \frac{1}{k^2}$. *[5]*

NOTE: The point of **2** is that the basic form of Chebyshev's Inequality can't really be improved upon unless you have additional information about the distribution of the random variable $X$.