Mathematics 1550H – Introduction to Probability
TRENT UNIVERSITY, Summer 2015

Assignment #4
Due on Monday, 20 July, 2015.
The bell curve strikes!

Recall that the *normal density function* with mean μ and standard deviation σ is:

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

Showing that this is really a probability density function, *i.e.* that $\int_{-\infty}^{\infty} \varphi(x)\, dx = 1$, requires tools beyond those available in first-year calculus. (In particular, because there is no formula for the antiderivative of $\varphi(x)$ in terms of the functions and algebraic operations that we commonly use, tricks from multivariate calculus are needed to get the job done.) Perversely, however, computing the expected value, variance, and standard deviation of a random variable X with $\varphi(x)$ as its density function *is* quite doable with first-year calculus.

Suppose, then, that X is a normal random variable with mean μ and standard deviation σ, that is, one with $\varphi(x)$ as its probability density function.

1. Compute $E(X)$. *[5]*

2. Compute $V(X)$ and check that $\sigma = \sqrt{V(X)}$. *[5]*

HINT: If you get anything other than $E(X) = \mu$ and $V(X) = \sigma^2$ after simplifying fully, something went wrong!