TRENT UNIVERSITY, SUMMER 2014

MATH 1550H Test

14 July, 2014

Time: 50 minutes

Name: ______________________________

STUDENT NUMBER: ______________________________

Question	Mark	
1	____	
2	____	
3	____	
Bonus	____	
Total	____	/30

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

Bonus. If A, B, and C are events in a sample space S such that A and B are independent and B and C are independent, must A and C be independent too? Why or why not? *[1]*

1. Do any *three* (3) of **a**–**d**. *[12 = 3 × 4 each]*

a. Four cards are drawn at random, in order with replacement, from a standard 52-card deck. What is the probability that exactly two of the four cards are $\diamondsuit$s?

b. A fair standard die is rolled once. Let E be the event that an even number comes up, and F be the event that three or six comes up. Determine whether E and F are independent or not.

c. A fair coin is tossed repeatedly. What is the probability that at least two tails come up before the first head occurs?

d. A fair standard die is rolled repeatedly. What is the expected number of rolls before six comes up for the second time?

2. Do any *two* (2) of **a**–**c**. *[10 = 2 × 5 each]*

a. A fair coin is tossed repeatedly until either the second head or the second tail occurs. Let Y be the number of tosses required. Compute $E(Y)$ and $V(Y)$.

b. Show that if A and B are any events in a sample space S and $0 < P(B) < 1$, then $P(A) = P(A|B)P(B) + P\big(A|\bar{B}\big)P\big(\bar{B}\big)$.

c. All ten of one red, two blue, three green, and four yellow balls are arranged in a row. How many arrangements are there if balls of the same colour cannot be told apart?

3. Do either *one* (1) of **a** or **b**. *[8]*

a. Three fair coins have identical heads (H), and two of the three that were minted in the same year have identical tails (T_1), but the third was minted in a different year and has a distinct tail (T_2). One of the three coins is selected at random, tossed, and replaced; this is repeated for a total of two tosses. Let Z be the number of T_1s plus two times the number of T_2s that come up. Compute the expected value and standard deviation of Z.

b. Initially, jar I contains one blue and two red balls, jar II contains one red and two blue balls, and jar III is empty. Two balls are chosen randomly, without replacement, from each of jars I and II and placed in jar III. Let X be the number of red balls that end up in jar III. Compute the expected value and standard deviation of X.

[Total = 30]