Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2014
Quizzes

Quiz #1. Wednesday, 25 June, 2014. *[15 minutes]*

A standard six-sided die is modified to have the number 1 appear on one face, 2 on two faces, and 3 on the remaining three faces. The die is otherwise fair.

0. If the die is thrown once, what is the probability of each number coming up? *[1]*

Now suppose the modified die is thrown twice, producing an ordered pair (a, b) of numbers, where a is the result of the first throw and b is the result of the second throw.

1. What is the sample space for this experiment? *[1]*
2. Determine the probability $P(F)$ of the event F that exactly one of the tosses gives 2. *[1.5]*
3. Determine the probability $P(E)$ of the event E that the sum of the two tosses is even. *[1.5]*

Quiz #2. Monday, 30 June, 2014. *[15 minutes]*

A hand of five cards is dealt out face up, one card at a time and without replacement, from a standard 52-card deck (A, K, Q, J, 10, 9, 8, 7, 6, 5, 4, 3, 2 of each of four suites, $\heartsuit$, $\diamondsuit$, $\clubsuit$, and $\spadesuit$).

1. How many such hands are there? *[1]*
2. How many hands are there in which all five cards are from the same suite? *[1]*

Wanda Maximoff enchants the deck so that every card drawn from it is instantly replaced with a perfect copy and the deck magically reshuffled before the next card is drawn. A hand of five cards is dealt out face down, one card at a time, from the enchanted deck, but the order the cards were drawn in is lost when they are turned face up, so order does not matter.

3. How many possible such hands are there? *[2]*
4. How many hands are there in which all five cards are from the same suite? *[1]*

Bonus. Who is Wanda Maximoff? *[0.1]*

Quiz #3. Wednesday, 2 July, 2014. *[15 minutes]*

1. Max is given fireworks for the holiday, seven distinct rockets and nine distinct Roman candles. [That is, you can tell different rockets, or different Roman candles, apart.] He proceeds to set them all off, one after another, in random order. If the fourth and fifth fireworks were both rockets, what is the probability that first three were all Roman candles? *[5]*

Quiz #4. Monday, 7 July, 2014. *[15 minutes]*

A jar contains the three balls inscribed with the numbers 1, 2, and 3, respectively. Two balls are randomly chosen, in order with replacement, from the jar. Let the random variable X be the sum of the numbers inscribed on the chosen balls.

1. Find the probability function of X. *[3]*
2. Compute the expected value, $E(X)$, of X. *[2]*

Quiz #5. Wednesday, 9 July, 2014. *[15 minutes]*

The roll of a fair die is declared to be a success if 3 or 6 comes up, and a failure otherwise. Let X be the number of successes in twelve (12) rolls of the die.

1. What kind of probability distribution does X have? *[1]*
2. Give the probability function of X. *[2]*
3. Compute the expected value, $E(X)$, and standard deviation, $\sigma(X)$, of X. *[2]*

Quiz #6. Wednesday, 16 July, 2014. *[15 minutes]*

Suppose the continuous random variable X has $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & x < 0 \text{ or } x > 1 \end{cases}$ as its probability density function.

1. Verify that $f(x)$ is indeed a probability density function. *[3]*
2. Compute the expected value $E(X)$ of X. *[2]*

Quiz #7. Monday, 21 July, 2014. *[15 minutes]*

Suppose X is a continuous exponential random variable with parameter $\theta > 0$, so it has density function $f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & x \geq 0 \\ 0 & x < 0 \end{cases}$.

1. Find the *median* of X, *i.e.* the m such that $P(X \leq m) = P(X > m) = \dfrac{1}{2}$, in terms of θ. *[5]*

Quiz #8. Wednesday, 23 July, 2014. *[10 minutes]*

1. Suppose X is a continuous random variable that has a normal distribution with parameters $\mu = 1$ and $\sigma = 2$. Use your table for the standard normal distribution to estimate the probability that $X < 0$. *[5]*

Quiz #9. Monday, 28 July, 2014. *[12 minutes]*

Suppose the discrete random variables X and Y are jointly distributed according to the following table:

$X \backslash Y$	0	1
1	0.1	0.2
2	0.3	0.1
3	0.1	0.2

1. Compute $\text{cov}(X, Y)$, the covariance between X and Y. *[4]*
2. Determine whether X and Y are independent or not. *[1]*

Quiz #10. Wednesday, 30 July, 2014. *[10 minutes]*

Suppose X and Y are random variables such that $E(X) = 1.5$, $V(X) = 0.25$, $E(Y) = -0.2$, $V(Y) = 0.96$, and $\text{cov}(X, Y) = -0.8$.

1. Let $U = 2X - Y$. Compute $E(U)$ and $V(U)$. *[5]*