Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2014

FINAL EXAMINATION
Friday, 6 August, 2014

Time: 3 hours *Brought to you by* Стефан Біланюк.

Instructions: Do both of parts ♡ and ♢, and, if you wish, part ♣. Show all your work and simplify answers as much as practicable. *If in doubt about something,* **ask!**

Aids: Calculator; one $8.5'' \times 11''$ or A4 aid sheet; standard normal table; $\leq 10^{10}!$ neurons.

Part ♡. Do all of **1**–**5**. *[Subtotal = 70/100]*

1. A fair standard six-sided die is rolled four times. Let X be the number of times that odd-numbered faces come up, and let Y be the sum of the odd-numbered faces that come up.

a. What is the probability function of X? *[5]*

b. Compute $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. *[5]*

c. Without computing it from scratch, what should $E(Y)$ be? Explain why. *[5]*

2. A continuous random variable Z is uniformly distributed between 0 and 3.

a. What is the expected value of Z^2? *[10]*

b. What is the probability that Z^2 is at most 8? *[5]*

3. A hand of five cards is drawn simultaneously (*i.e.* without order or replacement) from a standard 52-card deck.

a. What is the probability that the hand is a "full house," that is, that it includes a pair of one kind and a triple of another kind? *[8]*

b. What is the probability that all the cards in the hand are of different kinds? *[7]*

4. Suppose Y is a normally distributed continuous random variable with expected value $\mu = 3$ and standard deviation $\sigma = 2$.

a. Compute $P(Y \geq 0)$ using your standard normal table. *[5]*

b. Find the median of Y, *i.e.* the number m such that $P(Y \leq m) = 0.5$, with the help of your standard normal table. *[5]*

5. A fair coin is tossed, and then tossed some more until it comes up again with whatever face came up on the first toss.

a. What are the sample space and probability function? *[7]*

b. Let A be the event that no more than three tosses took place and let B be the event that the second toss was a head. Determine whether the events A and B are independent or not. *[8]*

[Parts ♢ and ♣ are on page 2.]

Part ♢. Do any *two* (2) of **6–10**. *[Subtotal = 30/100]*

6. Suppose Z is a continuous random variable with a standard normal distribution.

a. Compute $P(-2 < Z < 2)$ using your standard normal table. *[5]*

b. Use Tschebysheff's Inequality to show that $P(-2 \leq Z < 2) \geq \frac{3}{4}$. *[5]*

c. Without computing the relevant integral, explain why $\mathrm{E}(Z) = 0$. *[5]*

7. A fair standard six-sided die is rolled twice.

a. What is the probability of a sum of 6 for the two rolls, given that the two rolls gave different numbers? *[7]*

b. Determine whether the events that the two rolls are different and that the sum of the two rolls is 7 are independent or not. *[8]*

8. Suppose the discrete random variables X and Y are jointly distributed according to the following table:

$X \backslash Y$	0	1	2
1	0.2	0.1	0.1
2	0.1	0	0.1
3	0.1	0.1	02

a. Compute $E(X)$, $E(Y)$, $V(X)$, $V(Y)$, and $\mathrm{cov}(X, Y)$. *[10]*

b. Let $U = X - 2Y$. Compute $E(U)$ and $V(U)$. *[5]*

9. Suppose A, B, and C are events such that $P(ABC) = P(A)P(B)P(C)$. Does it follow that the first two events, A and B, are independent? Show that it does or give an example to show that it doesn't. *[15]*

10. Let $f(y) = \begin{cases} \frac{1}{3}y^2 & -1 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$ be the probability density function of the continuous random variable Y.

a. Verify that $f(y)$ is indeed a probability density function. *[7]*

b. Compute $E(Y)$ and $V(Y)$. *[8]*

[Total = 100]

Part ♣. Bonus!

K. If two fair standard six-sided dice are rolled together three times, what is the probability that the two dice will match (*i.e.* have the same number come up) on one of the three rolls? *[1.5]*

Q. Write an original little poem about probability or mathematics in general. *[1.5]*

I HOPE THIS WENT BETTER THAN THE TEST DID. :-)
ENJOY THE REST OF THE SUMMER!