Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2014

Assignment #5
Can we have Tchebysheff's equality?
Due on Monday, 28 July, 2014.

Recall from class and the textbook that the following is true:

TCHEBYSHEFF'S THEOREM. Suppose X is a random variable, discrete or continuous, with expected value $E(X) = \mu$ and standard deviation $\sigma(X) = \sigma$. Then, for any real number $k > 0$, $P\left(|X - \mu| < k\sigma^2\right) \geq 1 - \dfrac{1}{k^2}$.

Your task in this assignment, should you choose to accept it, will be to see if there are, under reasonable conditions, random variables X for which we actually get an equality instead, *i.e.* $P\left(|X - \mu| < k\sigma^2\right) = 1 - \dfrac{1}{k^2}$.

Do *one* (1) of problems **1** or **2** below. To keep things relatively simple, we will assume in both **1** and **2** that $\mu = 0$ and $\sigma = 1$, and also that X is symmetric about $\mu = 0$, *i.e.* that $P(0 \leq X \leq x) = P(-x \leq X \leq 0)$ for all $x \geq 0$.

1. Is there a discrete random variable X, satisfying the assumptions above, which takes on (not necessarily positive) integer values such that $P\left(|X| < k\right) = 1 - \dfrac{1}{k^2}$ for every positive integer k? If so, give an example; if not, explain why not. *[10]*

2. Is there a continuous random variable X, satisfying the assumptions above, such that $P\left(|X| < k\right) = 1 - \dfrac{1}{k^2}$ for every positive real number k? If so, give an example; if not, explain why not. *[10]*