Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2013
Solution to Assignment #2
Boxed in …*

The Wizard of Oz shows you three closed boxes, and tells you that two of them are empty and the remaining one contains the magical red shoes you need (and want!) to return home from Oz. You are allowed to choose one box and proceed to do so. The Wizard opens one of the other two boxes (*i.e.* not the one you picked), which turns out to be empty, and asks you if you wish to change your mind about which box to pick.

1. Should you stick with your original choice or change it to the other unopened box? Explain your reasoning fully. *[10]*

SOLUTION. You should change mind. This may seem a little counter-intuitive … Here's a deceptively simple argument for why you should change:

Initially, before the Wizard opens one of the other boxes, you have a $\frac{1}{3}$ probability of guessing correctly and a $\frac{2}{3}$ chance of getting it wrong and choosing an empty box. If you chose the correct box, the Wizard may choose either of the other two (and empty!) boxes to open; if you chose an empty box, the Wizard must open the other empty box. If you think about it, changing your mind fails to get you the box with the shoes *only* if you guessed right initially about which box that was. It follows that changing your mind succeeds with a probability of $\frac{2}{3}$ – *i.e.* the probability that you guessed wrong initially – and fails with a probability of $\frac{1}{3}$ – *i.e.* the probability that you guessed right initially. ∎

NOTE: I rather suspect that fact that changing your mind effectively exchanges the initial probabilities for success and failure is part of what makes this problem seem paradoxical. For much more about the problem, look up the "Monty Hall problem" online; in particular, Wikipedia has a pretty good article at `en.wikipedia.org/wiki/Monty_Hall_problem` .