Mathematics 1550H – Introduction to probability
TRENT UNIVERSITY, Summer 2013

Quizzes

Quiz #1. Wednesday, 26 June, 2013. *[15 minutes]*

A fair (standard six-sided) die is tossed twice. Let b and c be the outcomes of the first and second tosses respectively. Since the die is fair, any pair (b, c) is as likely to turn up as any other.

1. What is the sample space S, exactly? How many possible outcomes are there in S? *[1]*
2. If E is the event that $b = 1$, determine $P(E)$. *[2]*
3. If F is the event that $b^2 \geq 4c$, determine $P(E)$. *[3]*

Quiz #2. Wednesday, 3 July, 2013. *[15 minutes]*

A fair two-sided coin has a thick flat edge. If it is tossed, it can land heads up (H) or tails up (T) with equal likelihood, or land on edge (E), which occurs with a probability of $0.01 = \frac{1}{100}$. With repeated tosses, their *sum* is computed by counting each H as 1 and each E as $\frac{1}{2}$ (*i.e.* $H = 1$, $E = \frac{1}{2}$, and $T = 0$). The coin is tossed three times. Compute the probability of the following events:

1. The coin landed on edge twice in the three tosses. *[2]*
2. The sum of the three tosses is an even integer. *[4]*

Quiz #3. Monday, 8 July, 2013. *[15 minutes]*

You are pet-sitting three dogs and two cats. Two of the dogs and one cat are black, and the remaining dog and cat are white. When you let them out into the backyard, they go out one at a time in random order.

1. What is the probability that first two pets to go out were both black? *[2]*
2. If one of the the last two pets to go out was white and the other was black, what is the probability that first two to go out were both black? *[4]*

Quiz #4. Wednesday, 10 July, 2013. *[15 minutes]*

A fair tetrahedral (four-sided) die has faces numbered 1 through 4. The random variable X is the sum of the numbers obtained by rolling this die twice.

1. Determine the probability mass function of X. *[4]*
2. Use the probability mass function to help find $P(4 \leq X \leq 6)$. *[2]*

Quiz #5. Wednesday, 17 July, 2013. *[10 minutes]*

A fair icosahedral (*i.e.* 20-sided) die, with faces numbered 1 through 20, is rolled; the roll is a success if the number obtained is divisible by at least one of 4 or 5, and a failure otherwise. Let X be the number of successes in five rolls of the die.

1. Compute the expected value $E(X)$ of X. *[3]*
2. Compute the standard deviation of σ_X of X. *[3]*

Quiz #6. Monday, 22 July, 2013. *[15 minutes]*

Do *one* (1) of 1 or 2 below.

1. A fair die is rolled. The result is a success if the roll is 3 or 6, and is considered a failure otherwise. Let X be the number of rolls required for the first success to occur.
   a. What is the probability mass function of X? *[3]*
   b. Find the expected value $E(X)$ and standard deviation σ_X of X. *[3]*
2. Suppose the probability density function of the continuous random variable X is $f(x) = \begin{cases} \frac{1}{2}\cos(x) & -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ 0 & x < -\frac{\pi}{2} \text{ or } x > \frac{\pi}{2} \end{cases}$. Compute $P(0 \leq X \leq 2)$. *[6]*

Quiz #7. Wednesday, 24 July, 2013. *[15 minutes]*

1. Compute the expected value $E(X)$ and variance $\text{Var}(X)$ of the continuous random variable X with probability density function $f(x) = \begin{cases} 3x^{-4} & 1 \leq x \\ 0 & x < 1 \end{cases}$. *[6]*

Quiz #8. Monday, 29 July, 2013. *[15 minutes]*

Recall that a continuous exponential random variable X with parameter $\lambda > 0$ has density function $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$ and cumulative distribution function $F(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$.

1. Find the *median* of X, *i.e.* the m such that $P(X \leq m) = P(X > m) = \frac{1}{2}$, in terms of λ. *[6]*

Quiz #9. Wednesday, 31 July, 2013. *[15 minutes]*

Do *one* (1) of 1 or 2 below.

1. If X is a Poisson random variable with parameter $\lambda > 0$, such that $p(0) = P(X = 0) = \frac{1}{2}$ and $p(1) = P(X = 1) = \frac{1}{2}\ln(2)$, what is $p(2) = P(X = 2)$? *[6]*
2. Suppose X is a normal random variable with parameters $\mu = 2$ and $\sigma = 1$. Show that $P(-1 \leq X \leq 5) \geq \frac{8}{9}$. *[6]*