Mathematics 1550H – Introduction to probability
Trent University, Summer 2013

Final Examination
for practice!

Time: 3 hours *Brought to you by* Стефан Біланюк.

Instructions: Do both of parts ♠ and ♣, and, if you wish, part ♢. Show all your work. *If in doubt about something,* **ask!**

Aids: Calculator; one $8.5'' \times 11''$ or A4 aid sheet; $\leq 10!^{10!}$ neurons.

Part ♠. Do all of **1**–**5**. *[Subtotal = 70/100]*

1. Two numbers are chosen at random in succession, with replacement, from the set $\{1, 2, 3, \ldots, 100\}$. What is the probability that the first one is larger than the second one? *[10]*

2. In a set of dominoes, each piece is marked with two numbers, one on each end. The pieces are symmetrical, so that the two numbers are unordered. (That is, you can't tell $(1,4)$ and $(4,1)$ apart.) We will consider dominoes formed using the numbers $\{0, 1, \ldots, 12\}$ in what follows.

a. How many different dominoes are possible? *[5]*

b. If your set of dominoes has exactly one of each possible domino, what is the probability that the numbers on a randomly chosen domino add up to an odd number? *[10]*

3. Let X be a continuous random variable with a standard normal distribution.

a. Verify that $P(-2 \leq X \leq 2) \geq 0.75$. *[5]*

b. Compute $E(|X|)$. *[10]*

4. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} cx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

a. Determine the value of c. *[5]*

b. Find the cumulative distribution function, $F(x)$, of X. *[8]*

c. Find an interval $[a,b]$ such that $P(a \leq X \leq b) \geq 0.75$. *[2]*

5. A toaster factory expects to make five defective toasters a day. Suppose the number of defective toasters made in one day has a Poisson distribution with parameter $\lambda > 0$.

a. What is the parameter λ? *[2]*

b. What is the probability that no defective toasters are made in a given day? *[3]*

c. If the factory operates for 300 days in one year, what is the expected number of days on which no defective toasters were made in the course of that year? *[10]*

[Parts ♣ and ♢ are on page 2.]

Part ♣. Do any *two* (2) of **6**–**9**. *[Subtotal = 30/100]*

6. An urn contains 10 yellow marbles and 15 blue marbles. Marbles are chosen randomly from the urn until the 4th yellow marble turns up.

a. How many marbles would you expect to have to choose if each marble is replaced before the next is chosen? *[7]*

b. How many marbles would you expect to have to choose if marbles are not replaced before the next is chosen? *[8]*

7. Each of the members of a committee of seven independently makes a correct decision with a probability of 0.7. The committee as a whole makes decisions by majority rule.

a. What is the probability that the committee as a whole makes a given decision correctly? *[7]*

b. Given that the committee voted 4 to 3 to make a decision, what is the probability that it is correct?*[8]*

8. Suppose X is a continuous random variable with an exponential distribution with parameter $\lambda = 1$. Show that $E\left(X^k\right) = k!$ for each $k \geq 1$. *[15]*

9. Suppose X is a random variable with a distribution that is partly discrete and partly continuous: $P(X \leq t) = \begin{cases} 0 & t < 0 \\ \frac{1}{4}(1+t) & 0 \leq t \leq 3 \\ 1 & 3 < t \end{cases}$. (So, for example, $P(X = 0) = \frac{1}{4}$ even though $P(X < 0) = 0$, and $P(X \leq 2) = \frac{3}{4}$ even though $P(X = 2) = 0$.)

a. Compute $E(X)$. *[8]*

b. Compute $\mathrm{Var}(X)$. *[7]*

[Total = 100]

Part ♢. Bonus!

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∘∘‿. Write an original little poem about probability or mathematics in general. *[2]*

Enjoy the rest of the summer!