Mathematics 1350H – Linear algebra I: matrix algebra
Trent University, Summer 2017

Assignment #6
Due on or by Saturday, 17 June, at the exam.

A little linear optimization

Consider the set of points $(x, y, z) \in \mathbb{R}^3$ which satisfy all of the following conditions:

$$\begin{array}{lll} x \geq 0 & x \leq 12 & x+y+z \geq 12 \\ y \geq 0 & y \leq 12 & x+y+z \leq 24 \\ z \geq 0 & z \leq 12 & \end{array}$$

These points form a solid whose eight faces are triangles which are pieces of the planes $x = 0$, $y = 0$, $z = 0$, $x = 12$, $y = 12$, $z = 12$, $x+y+z = 12$, and $x+y+z = 24$, respectively.

1. Find all the vertices (*i.e.* corners) of the solid and draw a reasonably accurate sketch of the solid. *[4]*

2. For each of the following functions, find all the points of the solid on which the function achieves its maximum value on the solid.

a. $f(x, y, z) = x + y + z$ *[2]*

b. $g(x, y, z) = 2x + y + z$ *[2]*

c. $h(x, y, z) = x - 2y + z$ *[2]*

Note: A linear function, such as those aboves, will achieve its maximum on the solid at at least one vertex of the solid, though it may also achieve this maximum at points of this solid which are not vertices.