Mathematics 1350H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Summer 2017

Assignment #2

Orthogonalization

Due on Wednesday, 24 May, 2017.

The key to what follows is the following idea. The component of a vector $\mathbf{v}$ parallel to a (non-zero) vector $\mathbf{u}$ is the *projection of* $\mathbf{v}$ *onto* $\mathbf{u}$*:*

$$\mathrm{proj}_{\mathbf{u}}(\mathbf{v}) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\right)\mathbf{u}$$

Further, if you take away the component of $\mathbf{v}$ which is parallel to $\mathbf{u}$ away from $\mathbf{v}$, the remaining component, namely

$$\mathbf{v} - \mathrm{proj}_{\mathbf{u}}(\mathbf{v}) = \mathbf{v} - \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\right)\mathbf{u}\,,$$

is orthogonal to $\mathbf{u}$. (See §1.2 of the textbook for more on this.)

1. Suppose $\mathbf{v}$ and $\mathbf{u} \neq \mathbf{0}$ are both vectors in $\mathbb{R}^n$. Verify that $\mathbf{v} - \mathrm{proj}_{\mathbf{u}}(\mathbf{v})$ is orthogonal to $\mathbf{u}$. *[3]*

Hint: Use the dot product!

Now let $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ be a set of three vectors in $\mathbb{R}^3$. We will modify this set of vectors to make it nicer in some respects.

2. Use the idea in **1** to modify the second vector in B to make it orthogonal to the first vector in B. *[2]*

3. Modify the third vector in B to make it orthogonal to both the first and second vectors in B. *[3]*

4. Modify the first vector in B and your modified vectors from **2** and **3** to have length one. *[1]*

5. What might your final collection of modified vectors from **4** be good for? *[1]*