Mathematics 1350H – Linear Algebra I: Matrix Algebra

TRENT UNIVERSITY, Summer 2017

Assignment #1

Shifty business

Due on Monday, 15 May.

Suppose $\mathbf{a} = [a_1, a_2, \ldots, a_n]$ is an n-place row vector. The *left shift* of $\mathbf{a}$ by k places (where $0 \leq k < n$) is the vector $\sigma_k(\mathbf{a}) = [a_{k+1}, a_{k+2}, \ldots, a_n, a_1, a_2, \ldots, a_k]$. For example, here is a left shift by 2 places of a 6-place vector:

$$\sigma_2\left([3, -1, 0, 5, 1, -2]\right) = [0, 5, 1, -2, 3, -1]$$

Note that for any vector $\mathbf{a}$, $\sigma_0\left(\mathbf{a}\right) = \mathbf{a}$; that is, the left shift by 0 places does nothing to the vector.

1. Explain why $\sigma_k(s\mathbf{a}) = s\sigma_k(\mathbf{a})$ and $\sigma_k(\mathbf{a} + \mathbf{b}) = \sigma_k(\mathbf{a}) + \sigma_k(\mathbf{b})$ for any n-place vectors $\mathbf{a}$ and $\mathbf{b}$, integer k with $0 \leq k < n$, and scalar s. *[2]*

2. Is it true that $\sigma_k\left(\sigma_\ell(\mathbf{a})\right) = \sigma_{k+\ell}(\mathbf{a})$, assuming that $0 \leq k + l < n$ too? Explain why or why not. *[2]*

3. Suppose we identify the point (a, b) in the Cartesian plane with the row vector $[a, b]$. Geometrically, what does σ_1 do to the Cartesian plane? *[2]*

4. How does the left shift operator interact with the dot product? *[2]*

5. Find a vector $\mathbf{a}$ with as many places as you can such that each entry of $\mathbf{a}$ is either $+1$ or -1, and such that for every left shift by $k > 0$ places we have $|\mathbf{a} \cdot \sigma_k(\mathbf{a})| \leq 1$. *[2]*