Mathematics 1350H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Summer 2015

MATH 1350H Test
1 June, 2015
Time: 60 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Do any *two* (2) of **a**–**c**. *[10 = 2 × 5 each]*

Consider the lines given by the vector-parametric equations $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $t \in \mathbb{R}$, and $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + s \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $s \in \mathbb{R}$.

a. Find the angle between the lines.

b. Find an equation of the form $ax + by = c$ for each of the lines.

c. Find the point where the lines intersect.

2. Do any *two* (2) of **a**–**c**. *[10 = 2 × 5 each]*

Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$.

a. Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

b. Solve the equation $\mathbf{u} - 2\mathbf{v} + 3\mathbf{w} - 4\mathbf{x} = \mathbf{0}$ for $\mathbf{x}$.

c. Find the components of $\mathbf{w}$ that are, respectively, parallel to and perpendicular to $\mathbf{v}$.

3. Consider the following system of linear equations:

$$\begin{array}{rcrcrcl} 2x & + & y & + & z & = & 4 \\ x & + & 2y & + & z & = & 4 \\ x & + & y & + & 2z & = & 4 \end{array}$$

a. Find all the solutions, if any, of this system. *[8]*

b. Use your answer to **a** to determine whether the vectors $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ are linearly dependent or independent. *[2]*

4. As in **2**, let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$. In addition, let $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$.

a. Determine whether $\mathbf{x} \in \text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$. *[8]*

b. Determine whether $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly dependent or independent. *[2]*

[Total = 40]