Mathematics 1350H – Linear algebra I: Matrix algebra
TRENT UNIVERSITY, Summer 2015

Quizzes

Quiz #1. Wednesday, 13 May, 2015. *[10 minutes]*

1. Find the vector in $\mathbb{R}^2$ that would take you from the point $(1, -1)$ to the point $(2, 1)$ and sketch it. *[3]*
2. Find the vector in $\mathbb{R}^3$ of length 10 in the same direction as $\begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$. *[2]*

Quiz #2. Wednesday, 20 May, 2015. *[12 minutes]*

Consider the lines in $\mathbb{R}^3$ given by the vector equations $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $t \in \mathbb{R}$, and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $s \in \mathbb{R}$.

1. Find the point where the lines intersect. *[0.5]*
2. Find the angle between the lines *[2]*
3. Find an equation of the form $ax + by + cz = d$ of the plane that includes both lines. *[2.5]*

Quiz #3. Monday, 25 May, 2015. *[20 minutes]*

1. The following system of linear equations has exactly one solution. Use the Gauss-Jordan method to find it. Show all your work. *[5]*

$$\begin{array}{ccccccc} 2x & + & y & + & 3z & = & 2 \\ x & & & + & z & = & 1 \\ x & - & y & - & z & = & 2 \end{array}$$

Quiz #4. Wednesday, 27 May, 2015. *[20 minutes]*

1. Determine whether the vectors $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 7 \\ 8 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ are linearly dependent or independent. *[5]*

Quiz #5. Wednesday, 3 June, 2015. *[15 minutes]*

1. Find the inverse matrix of $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ or show that it does not have an inverse. *[5]*

Quiz #6. Monday, 8 June, 2015. *[15 minutes]*

Determine whether each of the following sets is a subspace of $\mathbb{R}^2$ or not.

1. $U = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \,\middle|\, 2x - y = 0 \right\}$ *[1.5]*
2. $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \,\middle|\, 2x - y = 13 \right\}$ *[1.5]*
3. $W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \,\middle|\, x^2 - y = 0 \right\}$ *[2]*

Take-Home Quiz #7. Due on Wednesday, 10 June, 2015. *[15 minutes]*

With apologies to Prof. Tolkien ...

If the Númenoreans had been mathematicians, perhaps the rhyme of lore* Gandalf quotes to Pippin during the ride from Rohan to Gondor in the *The Lord of the Rings* would have been something like:

Tall ships and tall kings
Three times three,
What brought they from the foundered land
Over the flowing sea?
Seven points and seven lines
In one geometry:
Every point met three lines,
Every line met points three,
Every pair of points connected,
Every line pair intersected.

1. Draw a picture of this alternate universe Númenorean geometry. *[5]*

Quiz #8. Wednesday, 10 June, 2015. *[15 minutes]*

1. Find a basis for the subspace $U = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \,\middle|\, \begin{array}{rcrcrcrcl} 2x & - & y & + & z & - & 2w & = & 0 \\ -x & + & 2y & + & z & + & w & = & 0 \\ x & + & y & + & 2z & - & w & = & 0 \\ 4x & + & y & + & 5z & - & 4w & = & 0 \end{array} \right\}$

   of $\mathbb{R}^4$. *[5]*

* "Tall ships and tall kings/ Three times three,/ What brought they from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree."

Quiz #9. Monday, 9 June, 2015. *[20 minutes]*

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 3 & 5 & 5 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$.

1. Apply the Gauss-Jordan algorithm to fully row-reduce $\mathbf{A}$. *[1]*
2. Use the results of your computation for question 1 to help find the following:
   a. The rank and nullity of $\mathbf{A}$. *[0.5]*
   b. Whether $\mathbf{A}$ is invertible or not. *[0.5]*
   c. A basis for the row space, row($\mathbf{A}$), of $\mathbf{A}$. *[1]*
   d. A basis for the column space, col($\mathbf{A}$), of $\mathbf{A}$. *[1]*
   e. A basis for the null space, null($\mathbf{A}$), of $\mathbf{A}$. *[1]*