

Mathematics 1350H – Linear algebra I: matrix algebra

TRENT UNIVERSITY, Summer 2015

ASSIGNMENT #5

Due on Monday, 15 June, 2015.

“What is the matrix?”

1. Find a 2×2 matrix \mathbf{X} such that $\mathbf{X}^2 = -\mathbf{I}_2$. [2]
2. Verify that if \mathbf{X} is the matrix you obtained in **1**, and $\mathbf{A} = a\mathbf{I}_2 + b\mathbf{X}$ for scalars a and b (not both 0), then \mathbf{A} is invertible and there are scalars d and c such that $\mathbf{A}^{-1} = d\mathbf{I}_2 + c\mathbf{X}$. [3]
3. Find 4×4 matrices \mathbf{U} , \mathbf{V} , and \mathbf{W} such that $\mathbf{U}^2 = \mathbf{V}^2 = \mathbf{W}^2 = -\mathbf{I}_4$, $\mathbf{UV} = \mathbf{W}$, $\mathbf{VU} = -\mathbf{W}$, $\mathbf{VW} = \mathbf{U}$, $\mathbf{WV} = -\mathbf{U}$, $\mathbf{WU} = \mathbf{V}$, and $\mathbf{UW} = -\mathbf{V}$. [2]
4. Verify that if \mathbf{U} , \mathbf{V} , and \mathbf{W} are the matrices you obtained in **3**, and $\mathbf{B} = a\mathbf{I}_4 + b\mathbf{U} + c\mathbf{V} + d\mathbf{W}$ for scalars a , b , c , and d (not all 0), then \mathbf{B} is invertible and there are scalars p , q , r , and s such that $\mathbf{B}^{-1} = p\mathbf{I}_4 + q\mathbf{U} + r\mathbf{V} + s\mathbf{W}$. [3]