Assignment #3
Due on Monday, 1 June, 2015.

Linear optimization

In this assignment we will deal with the solid whose faces are (parts of) the planes given by the equations $x = 0$, $y = 0$, $z = 0$, $x + 2y + 2z = 20$, $2x + y + 2z = 20$, $2x + 2y + z = 20$, and $x + y + z = 12$. Another way to look at this solid is as the set of points with coordinates (x, y, z) which satisfy *all* of the following seven inequalities: $x \geq 0$, $y \geq 0$, $z \geq 0$, $x + 2y + 2z \leq 20$, $2x + y + 2z \leq 20$, $2x + 2y + z \leq 20$, and $x + y + z \leq 12$.

1. Find the coordinates of all of the vertices of this solid and make as accurate a sketch as you can of it. *[6]*

2. Find the maximum value of the function $f(x, y, z) = 2x - y + z$ on this solid and determine at which point(s) of the solid this maximum occurs. *[4]*

Note: In this context the inequalities defining the solid are called *linear constraints*. Problems involving the optimization of a linear function subject to linear constraints arise often enough to be pretty important in the real world. If you're interested in the methods used to solve such problems, you might consider taking MATH-COIS 3350H, for which the only prerequisite is MATH 1350H.

Jennet ... Poor father. In the end he walked
    In Science like the densest night. And yet
    He was greatly gifted.
    When he was born he gave an algebraic
    Cry; at one glance measured the cubic content
    Of that ivory cone his mother's breast
    And multiplied his appetite by five.
    So he matured by a progression, gained
    Experience by correlation, expanded
    Into a marriage by contraction, and by
    Certain physical dynamics
    Formulated me. And on he went
    Still deeper into the calculating twilight
    Under the twinkling of five-pointed figures
    Till Truth became for him the sum of sums
    And Death the long division. My poor father.
    What years and powers he wasted.
    He thought he could change the matter of the world
    From the poles to the simultaneous equator
    By strange experiment and by describing
    Numerical parabolas.

       From *The Lady's Not For Burning*, by Christopher Fry.
       (My favourite play! Стефан)