Mathematics 1350H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Summer 2015

ASSIGNMENT #2
Due on Monday, 25 May, 2015.

Projection and Orthogonalization

The key to what follows is the following idea. Recall from §1.2 of the textbook that the component of a vector $\mathbf{v}$ parallel to a (non-zero) vector $\mathbf{u}$ is the *projection of* $\mathbf{v}$ *onto* $\mathbf{u}$*:*

$$\text{proj}_{\mathbf{u}}(\mathbf{v}) = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}}\right) \mathbf{u}$$

Recall further that if you take away the component of $\mathbf{v}$ which is parallel to $\mathbf{u}$ away from $\mathbf{v}$, the component that is left is orthogonal to $\mathbf{u}$.

1. Suppose $\mathbf{v}$ and $\mathbf{u} \neq \mathbf{0}$ are vectors of the same dimension. Verify that $\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})$ is orthogonal to $\mathbf{u}$. *[2]*

Hint: Use the dot product!

Now let $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$ be a set of three vectors in $\mathbb{R}^3$. We will modify this set of vectors to make it nicer in some respects.

2. Use the idea in **1** to modify the second vector in B to make it orthogonal to the first vector in B. *[2]*

3. Modify the third vector in B to make it orthogonal to both the first and second vectors in B. *[2]*

4. Modify the first vector in B and your modified vectors from **2** and **3** to have length one. *[2]*

5. What might your final collection of modified vectors from **4** be good for? *[2]*