Mathematics 1350H – Linear algebra I: Matrix algebra
Trent University, Summer 2014

Solutions to Assignment #1
Two classic puzzles

The two questions below are similar to problems posed in the Middle Ages, and may well go back farther than that.

1. Three men robbed a gentleman of a vase, containing 24 ounces of balsam. Whilst running away they met in a wood with a glass-seller, of whom in a great hurry they purchased three vessels. On reaching a place of safety they wished to divide the booty, but they found that their vessels contained 5, 11, and 13 ounces respectively. How could they divide the balsam into equal portions? (Explain in detail, please!) *[5]*

Solution. Here is the solution given in the source [1] from which I got both problems:

> Problems like this can be worked out only by trial: there are several solutions, of which one is as follows.

The vessels can contain	24 oz.	13 oz.	11 oz.	5 oz.
Their contents originally are	24...	13...	11...	5...
First, make their contents ...	0...	8...	11...	5...
Second, ” ” ...	16...	8...	0...	0...
Third, ” ” ...	3...	8...	8...	5...
Fourth, ” ” ...	3...	8...	8...	5...
Fifth, ” ” ...	3...	8...	8...	5...
Sixth, ” ” ...	8...	8...	8...	0...

A little effort should suffice to work out what pouring is done between containers at each step. □

2. A game is played by two people, say A and B. A begins by mentioning some number not greater than six, B may add to that any number not greater than six, A may add to that again any number not greater than six, and so on. The winner is the first to reach fifty. Assuming both A and B play as well as possible, which one should win? Explain why in detail. *[5]*

Solution. Here is the solution given in the source [1] from which I got both problems:

> ... Obviously, if A calls 43, then whatever B adds to that, A can win next time. Similarly, if A calls 36, B cannot prevent As calling 43 the next time. In this way it is clear that the key numbers are those forming the arithmetical progression 43, 36, 29, 22, 15, 8, 1; and whoever plays first ought to win.
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> Similarly, if no number greater than m may be added at any one time, and n is the number to be called by the victor, then the key numbers will be those forming the arithmetical progression whose common difference is $m+1$ and whose smallest term is the remainder obtained by dividing n by $m+1$.

Again, a little reflection and/or experimentation should suffice to convince you just how this solution works. ■

Reference

1. *Mathematical Recreations and Essays* (Fourth Edition), W.W. Rouse Ball, MacMillan and Co., London, 1896. [The two problems and their solutions are on pages 16–17 of Project Gutenberg's electronic version of this book, for which see: `http://www.gutenberg.org/ebooks/26839`]