Mathematics 1350H – Linear algebra I: Matrix algebra
TRENT UNIVERSITY, Summer 2013

MATH 1350H Test
3 June, 2013
Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator and an aid sheet.

1. Consider the line in $\mathbb{R}^3$ given by the vector equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

a. Find two points on this line. *[1]*

b. Sketch this line. *[2]*

c. Find a vector perpendicular to this line. *[3]*

d. Find the angle between this line and the line given by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. *[4]*

2. Consider the following system of linear equations:

$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 3 \\ x & - & y & + & z & = & 1 \\ x & + & 3y & + & z & = & k \end{array}$$

a. Find the solution(s), if any, of this system of equations if $k = 2$. *[5]*

b. Find the solution(s), if any, of this system of equations if $k = 5$. *[5]*

3. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Find a linear equation for the plane given by the vector-parametric equation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}.$$

b. Find a vector-parametric equation for the plane $2x - y + z = 2$.

c. Find the point(s) of intersection, if any, of the lines in $\mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $x + y = -1$, respectively.

4. Do any *two (2)* of **a**–**c**. *[10 = 2 × 5 each]*

a. Compute $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \end{bmatrix}$.

b. If $\mathbf{A}^T\mathbf{B} = \mathbf{I}_{41}$ for some matrices $\mathbf{A}$ and $\mathbf{B}$, what is $\mathbf{B}^T\mathbf{A}$?

c. If $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$, find the vector $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ such that $\mathbf{Ax} = \mathbf{b}$.

[Total = 40]