Mathematics 1350H – Linear algebra I: Matrix algebra

Trent University, Summer 2013

Solution to Assignment #1

Please see Assignment #1 for the story *Petty Cash* in which Clara is asked to solve a mathematical problem.

1. Clara is asked to solve a mathematical problem. Solve it, and give a complete explanation of your solution. *[10]*

Solution. *Petty Cash* is Knot VII of Lewis Carroll's *A Tangled Tale*, which was originally serialized in a magazine. Here's the author's solution:

Problem.—Given that one glass of lemonade, 3 sandwiches, and 7 biscuits, cost $1s.$ $2d.$; and that one glass of lemonade, 4 sandwiches, and 10 biscuits, cost $1s.$ $5d$: find the cost of (1) a glass of lemonade, a sandwich, and a biscuit; and (2) 2 glasses of lemonade, 3 sandwiches, and 5 biscuits.*

Answer.—(1) $8d.$; (2) $1s.$ $7d.$

Solution.—This is best treated algebraically. Let x = the cost (in pence) of a glass of lemonade, y of a sandwich, and z of a biscuit. Then we have $x + 3y + 7z = 14$, and $x + 4y + 10z = 17$. And we require the values of $x + y + z$ and of $2x + 3y + 5z$. Now, from *two* equations only, we cannot find, *separately*, the values of *three* unknowns: certain *combinations* of them may. however, be found. Also we know that we can, by the help of the given equations, eliminate 2 of the 3 unknowns from the quantity whose value is required, which will then contain one only. If, then, the required value is ascertainable at all, it can only be by the 3rd unknown vanishing of itself: otherwise the problem is impossible.

Let us then eliminate lemonade and sandwiches, and reduce everything to biscuits—a state of things even more depressing than "if all the world were apple-pie"—by subtracting the 1st equation from the 2nd, which eliminates lemonade, and gives $y + 3z = 3$, or $y = 3 - 3z$; and then substituting this value of y in the 1st, which gives $x - 2z = 5$, *i.e.* $x = 5 + 2z$. Now if we substitute these values of x, y, in the quantities whose values are required, the first becomes $(5 + 2z) + (3 - 3z) + z$, *i.e.* 8: and the second becomes $2(5 + 2z) + 3(3 - 3z) + 5z$, *i.e.* 19. Hence the answers are (1) $8d.$, (2) $1s.$ $7d.$

Yes, this is the same Lewis Carroll who wrote *Alice in Wonderland*. He was a mathematician, a poet, and a photographer, as well as a writer. ■

* A note for American [and modern] readers: Knot VII. In British currency, a shilling contains twelve pence. The phrase "One and two-pence" (written 1 *s.* 2 *d.*) means "one shilling and two-pence."