Mathematics 1350H – Linear algebra I: matrix algebra
Trent University, Fall 2009

Assignment #6
Due on Friday, 11 December, 2009.

A t0y un1verse

We're going to define our vectors using a different bunch of scalars, namely $\mathbb{Z}_2 = \{0, 1\}$, where $+$ and $\cdot$ are given by the following tables:

$+$	0	1
0	0	1
1	1	0

$\cdot$	0	1
0	0	0
1	0	1

If you know about modular arithmetic, this is just addition and multiplication modulo 2. You may take it on faith that this gives something algebraically well-behaved enough to be usable as a set of scalars.

The set of three-dimensional vectors we get from these scalars is

$$\mathbb{Z}_2^3 = \left\{ \begin{bmatrix} u \\ v \\ w \end{bmatrix} \,\middle|\, \text{each of } u,\ v,\ w \text{ is 0 or 1} \right\},$$

with addition of vectors and multiplication by scalars given by

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} a+r \\ b+s \\ c+t \end{bmatrix} \qquad \text{and} \qquad \alpha \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \alpha \cdot u \\ \alpha \cdot v \\ \alpha \cdot w \end{bmatrix},$$

using the addition and multiplication of scalars defined above.

1. How many vectors are there in $\mathbb{Z}_2^3$? List them all! *[2]*

2. If $\mathbf{u} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is a vector of $\mathbb{Z}_2^3$, what is $-\mathbf{u}$? Why? *[2]*

3. How many subspaces does $\mathbb{Z}_2^3$ have? List them all! *[4]*

4. Find an example of weird behaviour by the dot product of vectors in $\mathbb{Z}_2^3$. *[2]*