Mathematics 1350H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2008

SOLUTIONS TO ASSIGNMENT #1

1. Here is the author's own solution:

Let x be the number of pennies each had at first.

No. (3) received x, took out $(2+4)$, and put in $\frac{x}{2}$; so that the sack then contained $x \cdot \frac{3}{2} - 6$. Let us write 'a' for '$\frac{3}{2}$'.

No. (5) received $xa - 6$, took out $(4+1)$, and put in enough to multiply, by a, its contents when he received it. The sack now contained $(xa^2 - 6a - 5)$.

No. (2) took out $(1+3)$, and handed on $(xa^3 - 6a^2 - 5a - 4)$.

No. (4) took out $(3+5$, and handed on $(xa^4 - 6a^3 - 5a^2 - 4a - 8)$.

No. (1) put in 2. The sack now contained $5x$.

Hence $xa^4 - 6a^3 - 5a^2 - 4a - 6 = 5x$;

$$\begin{aligned}
\Leftrightarrow x &= \frac{6a^3 + 5a^2 + 4a + 6}{a^4 - 5} \\
&= \frac{6 \cdot 3^3 + 5 \cdot 3^2 \cdot 2 + 4 \cdot 3 \cdot 2^2 + 6 \cdot 2^3) \cdot 2}{3^4 - 5 \cdot 2^4} \\
&= \frac{(162 + 90 + 48 + 48) \cdot 2}{81 - 80} = 696 = \mathit{2l.\ 18s.\ 0d.}
\end{aligned}$$

Q.E.F.

It's obvious calculators weren't that common then ... One question: What's with the "Q.E.F." at the end? ("Q.E.D." you've probably heard of.) ■

Note. Charles Lutwidge Dodgson, better known under his pen name of Lewis Carroll, devised the problem. (It's problem number 52 from his book *Pillow Problems*.) These days he is remembered mainly for two books for children (and lots of adults!), *Alice in Wonderland* and *Through the Looking Glass*, and some nonsense verse, especially *Jabberwocky* and *The Hunting of the Snark*.

2. Ceilidh must use at least six cuts, even if she tries to rearrange the pieces at intermediate stages. To see this observe that the center cube of the 27 small ones the larger cube is cut up into has – like every other cube – six faces. Each of those six faces requires a separate cut, because there is no way to rearrange the pieces at any stage to allow two or more faces to result fro a single cut. ■