Mathematics 1350H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2008

Extra Credit Assignments
Due on Friday, 19 December, 2008.

You may do either or both of the following assignments. *You may not work with others on these assignments, nor consult any sources other than your text, notes, and handouts in this class.* You may use each of these assignments to either

i. add to your pool of assignments from which the best four are counted,
ii. add to your pool of quizzes from which the best eight are counted, or,
iii. for at most one of the two, add your score (out of 10) on the assignment to your test mark (out of 40) up to a maximum total of 24/40.

Indicate which option which you wish to exercise on your solution to each assignment. If you do not indicate an on your solution, option *i* will be assumed.

Assignment α

Before you tackle this assignment, you might want to look through §5.1–5.3 in the text, which cover what is done on this assignment and much more besides.

1. Suppose $\mathbf{v}$ and $\mathbf{u} \neq \mathbf{0}$ are vectors in $\mathbb{R}^n$. Show that $\mathbf{v} - \mathrm{proj}_{\mathbf{u}}(\mathbf{v})$ is orthogonal to $\mathbf{u}$. *[2]*

Let $S = \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

2. Find a basis for S. *[1]*

3. Use the idea in **1** to modify the second vector in your basis for S to make it orthogonal to the first vector in the basis, modify the third vector in the basis to make it orthogonal to the first two, and so on until you're out of basis vectors. *[3]*

Note: This process is called *Gram-Schmidt orthogonalization.*

4. Explain, in detail, why the set of modified (except for the first one!) vectors you got in **3** is still a basis for S. *[3]*

5. Modify your new basis for S to make every vector in it be of length 1. *[1]*

A basis in which every basis vector is of unit length and orthogonal to every other basis vector is said to be *orthonormal*. These are the nicest bases!

Assignment β

1. Answer the following puzzle, devised by Hubert Phillips. Give all your reasoning. *[5]*

Mrs. Inkpen

Mr. Pisistratus Patriarch lived up to his somewhat unusual name. He had nine children, and no fewer than 31 grandchildren. In his will he left an exact number of dollars to each grandchild. Each girl was to get \$7 more than each boy. All 31 grandchildren were alive when Patriarch died, and their legacies totaled \$470. Of this amount, \$74 went to Mrs. Inkpen's children (she was Patriarch's eldest daughter). How many daughters had Mrs. Inkpen?

2. What should the next number in the sequence 1, 2, 6, 12, 60, 420, 840, ... be? *[5]*