Mathematics 1350H – Linear algebra I: matrix algebra

Assignment #1
Due on Friday, 19 September, 2008.

Beggars and Cubes

Five beggars sat down in a circle, and each piled up, in a heap before him, the pennies he had received that day: and the five heaps were equal.

Then spake the eldest and wisest of them, unfolding, as he spake, an empty sack.

"My friends, let me teach you a pretty little game! First, I name myself 'Number One', my left-hand neighbour 'Number Two,' and so on to 'Number Five.' I then pour into this sack the whole of my earnings for the day, and hand it on to him who sits next but one on my left, that is 'Number Three.' *His* part in the game is to take out of it and give to his two neighbours, so many pennies as represent their names (that is, he must give four to 'Number Four' and two to 'Number Two'); he must then put *into* the sack half as much as it ccontained when he received it; and he must then hand it on just as I did, that is, he must hand it to him who sits next but one on his left—who will of course be 'Number Five.' *He* must proceed in the same way, and hand it on to 'Number Two,' from whom the sack will find its way to 'Number Four,' and so to me again. If any player cannot furnish, from his own heap, the whole of what he has to put into the sack, he is at liberty to draw upon any of the other heaps, *except mine*!"

The other beggars entered into the game with much enthusiasm: and in due time the sack returned to 'Number One,' who put into it the two pennies he had received during the game, and carefully tied up the mouth of it with a string. Then, remarking "it is a *very* pretty little game," he rose to his feet, and hastily quitted the spot. The other four beggars gazed at each other with rueful countenances. Not one of them had a penny left!

1. How much did each beggar have at the beginning? *[8]*

Ceilidh Carpenter wishes to use a saw to cut a wooden cube, with sides 30 *cm* long, into 27 cubes, each with sides 10 *cm* long. Ceilie can do this pretty easily by making six cuts through the cube, keeping the pieces together in the cube shape.

2. Can Ceilidh reduce the number of necessary cuts by rearranging the pieces after each cut? If so, how? If not, why not? *[2]*