Mathematics 135H – Linear algebra I: matrix algebra
TRENT UNIVERSITY, Fall 2007

Assignment #6 – Extra!
Due on Thursday, 20 December, 2007.

Basic niceness

This is an extra assignment which you may do if you have missed more than one assignment in the class, or if you simply want a shot at outdoing an imperfect mark on some previous assignment. Before you tackle this assignment, you might want to look through §5.1–5.3 in the text, which cover what is done on this assignment and much more besides.

The key to what follows is the following idea. Recall from §1.2 that the component of a vector $\mathbf{v}$ parallel to a (non-zero) vector $\mathbf{u}$ is the *projection of* $\mathbf{v}$ *onto* $\mathbf{u}$*:*

$$\mathrm{proj}_{\mathbf{u}}(\mathbf{v}) = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{u}\cdot\mathbf{u}}\right)\mathbf{u}$$

What happens if you take away the component of $\mathbf{v}$ which is parallel to $\mathbf{u}$ away from $\mathbf{v}$?

1. Suppose $\mathbf{v}$ and $\mathbf{u} \neq \mathbf{0}$ are vectors in $\mathbb{R}^n$. Show that $\mathbf{v} - \mathrm{proj}_{\mathbf{u}}(\mathbf{v})$ is orthogonal to $\mathbf{u}$. (*Hint:* Use the dot product!) *[2]*

Now let $S = \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$; then S is a subspace of $\mathbb{R}^4$. We will build a very nice basis for S over the remaining problems.

2. Find a basis for S. *[1]*

We will modify this basis to make it nicer.

3. Use the idea in **1** to modify the second vector in your basis for S to make it orthogonal to the first vector in the basis, modify the third vector in the basis to make it orthogonal to the first two, and so on until you're out of basis vectors. *[3]*

Note: This process is called *Gram-Schmidt orthogonalization.*

4. Explain, in detail, why the set of modified (except for the first one!) vectors you got in **3** is still a basis for S. *[3]*

We now have a basis for S in which every basis vector is orthogonal to every other basis vector. We can make it even nicer:

5. Modify your new basis for S to make every vector in it be of length 1. *[1]*

A basis in which every basis vector is of unit length and orthogonal to every other basis vector is said to be *orthonormal*. These are the nicest bases!