**Mathematics 135H – Linear algebra I: matrix algebra**
TRENT UNIVERSITY, Fall 2007

**Assignment #5**
Due on Friday, 7 December, 2007.

**Rotations in $\mathbb{R}^3$**

Before you tackle this assignment, you should read §3.6 in the text and do the exercises from this section recommended in Homework Set #3. Note that most of the concrete examples and exercises in this section stick to $\mathbb{R}^2$. This assignment is concerned with extending some of the material in §3.6 on rotations about the origin in $\mathbb{R}^2$ to rotations about lines through the origin in $\mathbb{R}^3$.

**1.** Find the matrix $R^z_\theta$ of a rotation through an angle of $\theta$ about the $z$-axis. *[1]*

*Note:* This rotation leaves the $z$-coordinate unchanged. As with rotations about the origin in $\mathbb{R}^2$, $\theta$ is measured counterclockwise, starting with the positive $x$-axis, when the $xy$-plane is viewed from above (*i.e.* from the positive $z$-axis).

**2.** Find the matrix $R^y_\phi$ of a rotation through an angle of $\phi$ about the $y$-axis. *[1]*

*Note:* This rotation leaves the $y$-coordinate unchanged. The angle $\phi$ should be measured counterclockwise, starting with the positive $x$-axis, when the $xz$-plane is viewed from the positive $y$-axis.

**3.** Find the matrix $R^x_\alpha$ of a rotation through an angle of $\alpha$ about the $x$-axis. *[1]*

*Note:* This rotation leaves the $x$-coordinate unchanged. The angle $\alpha$ should be measured counterclockwise, starting with the positive $y$-axis, when the $yz$-plane is viewed from the positive $x$-axis.

**4.** Find a combination of the rotations you obtained in **1**–**3** that moves the $x$-axis onto the line through the origin with direction vector $[\,1 \quad 1 \quad 1\,]$. *[2]*

**5.** Find a combination of the rotations you obtained in **1**–**3** that moves the line through the origin with direction vector $[\,1 \quad 1 \quad 1\,]$ onto the $x$-axis. *[1]*

**6.** Find the matrix $R$ of a rotation through an angle of $\omega$ about the line through the origin with direction vector $[\,1 \quad 1 \quad 1\,]$. *[4]*

*Note:* The angle $\omega$ should be measured counterclockwise when the plane $x + y + z = 1$ is viewed from the first octant.

*Hint:* Put together **3**–**5**.

**Sylvester's Theorem**

A mathematician, Sylvester,
Had a wife he would often pester,
"As I raised the rank
All my null spaces shrank."
"Add them!" she said, so he kissed her.

*Stefan Bilaniuk*