

Mathematics 1121H – Calculus II

TRENT UNIVERSITY, Winter 2026

Assignment #9

Series Business IV

Due on Friday, 20 March.*

Recall from Assignment #6 that the harmonic series, $\sum_{k=1}^{\infty} \frac{1}{k}$, diverges; and recall from Assignment #4 that the alternating harmonic series, $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$, converges.

1. Determine whether the harmonic series given a repeated $++--$ pattern of signs,

$$1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \frac{1}{10} - \frac{1}{11} - \frac{1}{12} + \dots,$$

converges or not. [3]

2. Find a_n for each $n \geq 0$ such that $\sum_{n=0}^{\infty} a_n x^n = \left(\frac{1}{1-x}\right)^2$ when the series converges. [3]

HINT. What series is $\frac{1}{1-x}$ the sum of?

3. Let $\mathbb{M} = \{n \in \mathbb{N} \mid n \text{ is not divisible by } 2, 3, \text{ or } 5\}$. Show that $\sum_{m \in \mathbb{M}} \frac{1}{m}$ diverges. [1]

HINT. $30k + 1 \in \mathbb{M}$ for every integer $k \geq 0$. Note that the given series is the harmonic series with a lot of the terms simply omitted.

4. What is your favourite series? Explain why in a short poem. [1]

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.