Mathematics 1121H – Calculus II
Trent University, Winter 2026

Assignment #8
~~Series~~ Sequence Business I~~V~~
Due on Friday, 6 March.[*]

Recall from Assignment #6 that the sequence of partial sums of the harmonic series, $H_n = \sum_{k=1}^{n} \frac{1}{k}$, diverges, *i.e.* $\lim_{n\to\infty} H_n = \infty$. This assignment is about the related sequence $\gamma_n = H_n - \ln(n)$, which converges.

1. Write a SageMath subroutine that plots γ_n as an area for $n \geq 2$. *[4]*

Hint. $\ln(n) = \int_1^n \frac{1}{x}\,dx$.

2. Explain why $\gamma_n = \int_1^n \left(\frac{1}{\lfloor x \rfloor} - \frac{1}{x} \right) dx$ for $n \geq 2$, where $\lfloor x \rfloor = \text{floor}(x)$ is the greatest integer $\leq x$. *[2]*

Hint. What is $\lfloor x \rfloor$ for $k \leq x < k+1$?

3. Show – by hand! – that $\gamma = \lim_{n\to\infty} \gamma_n$ exists, and that $0.5 < \gamma < 1$. *[4]*

Hint. Show that $\frac{1}{2}\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \cdots\right] < \gamma < 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \cdots$. The Monotone Convergence Theorem may also come in handy.

Bonus. Verify by hand that $\gamma = \lim_{n\to\infty} \gamma_n = -\int_0^\infty e^{-x} \ln(x)\,dx$, showing all the essential steps. *[2]*

The Arithmetic Of Co-operation

When you're adding up committees
there's a simple rule of thumb:
that talents make a difference,
but follies make a sum.

Another grook by Piet Hein. Probably my favourite!

[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.