Mathematics 1121H – Calculus II

TRENT UNIVERSITY, Winter 2026

Assignment #7

Series Business III

Due on Friday, 6 March.[*]

This assignment is concerned with the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$, where $n! = n(n-1)(n-2)\cdots 2\cdot 1$ for positive integers n and $0! = 1$, mainly to make formulas like the one for series work without having to write exceptions when $n = 0$.

1. What is the sum of this series? *[1]*

HINT. Ask SageMath!

2. Prove that the given series is equal to the function obtained in answering question **1** by showing that they both satisfy the differential equation $\dfrac{dy}{dx} = y$ with initial condition $y = 1$ when $x = 0$. *[3]*

NOTE. We're exploiting the fact that there is only one solution to a given differential equation with given initial conditions. Proving that is a bit beyond the scope of this course ...

3. What is a_n in terms of n if $\displaystyle\sum_{n=0}^{\infty} a_n x^n = \left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right)^2$? *[2]*

4. Show that $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges no matter what the value of x is. *[4]*

HINT. If x is negative, the series is an alternating series; if x is positive, you can, among other possibilities, use the Monotone Convergence Theorem.

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[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.