**Mathematics 1121H – Calculus II**

Trent University, Winter 2026

**Assignment #6**

**Series Business II**

*Due on Friday, 27 February.**

This assignment is concerned with the non-alternating counterparts of the series considered in Assigment #4, which work out differently.

First, consider the *harmonic series*, $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$. This series does not *converge*, *i.e.* add up to a real number, which in this case means that it *diverges*, *i.e.* adds up to infinity. Despite its failure to add up to a real numer, this series and its partial sums arise in many areas of mathematics, enough so that the partial sums have their own name, the *harmonic numbers*, and notation $\displaystyle H_n = \sum_{k=1}^{n} \frac{1}{k}$.

**1.** Use a suitable `while` loop in SageMath to discover the least value of $n$ required to ensure that $\displaystyle H_n = \sum_{k=1}^{n} \frac{1}{k} \geq a$ for each of $a = 3$, 6, 9, 12. *[2]*

**2.** Give an informal proof that the harmonic series diverges. *[2]*

Second, consider the series that adds up the reciprocals of all the squares of positive integers, $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$. (This series seems to lack a commonly accepted name; your instructor likes to call it the *square harmonic series*.) Unlike the harmonic series, it is convergent: it sums to $\displaystyle\frac{\pi^2}{6}$, a fact first proved by Leonhard Euler (1707-1783) in his solution to what came to be called the *Basel problem*.

**3.** Use a suitable `while` loop in SageMath to discover the least value of $n$ required to ensure that $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} \geq \frac{\pi^2}{6} - \frac{1}{a}$ for each of $a = 5$, 40, 100, 1000. *[2]*

**4.** What pattern do you see in your solution to question **3**? Does this pattern hold for all positive integers $a$? If so, try to explain why; if not, give an example where the pattern fails. *[2]*

Hint. At minimum, some experimentation with other values of $a$ besides those asked for in question **3** is in order. Probably some research of the "look it up" variety, too.

**5.** Given that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$, what does $\displaystyle\sum_{i=1}^{\infty} \frac{1}{(2i-1)^2} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots$ add up to? Explain why. *[2]*

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.