Mathematics 1121H – Calculus II
TRENT UNIVERSITY, Winter 2026

Assignment #10
A Choice and an Approximation
*Due on Friday, 27 March.**

Instructions. Do question **1** and *one* (1) of questions **2** or **3**.

The *Taylor polynomial of degree $n \geq 0$ at a* of a function $f(x)$ is

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k .$$

Recall that the average value of a function $f(x)$ over an interval $[b, c]$ is $\frac{1}{c-b}\int_b^c f(x)\,dx$.

1. What is the least n such that the Taylor polynomial of degree n at 0 of $\sin(x)$ makes $T_n(x) - \sin(x)$ have an average value over $[0, \pi]$ that is between -0.1 and 0.1? *[6]*

HINT. SageMath has a handy `taylor` operator that computes the Taylor polynomial of degree n at a point a of any suitably differentiable function.

Recall that $\sum_{n=0}^{\infty} a_n$ is *absolutely convergent* if $\sum_{n=0}^{\infty} |a_n|$ is convergent. Absolutely convergent series must be convergent. Series that are convergent but not absolutely convergent are said to be *conditionally convergent*.

2. Explain why a conditionally convergent series can be made to add to any value between $-\infty$ and ∞ inclusive by rearranging the terms of the series, without discarding any of the terms or adding new ones. *[4]*

HINT. A conditionally convergent series must still pass the Divergence Test. Also, it must have infinitely many positive terms and infinitely many negative terms. (Why?) The positive terms must add up to ∞ and the negative terms must add up to $-\infty$. (Why?)

Recall from class that the Root Test is the following assertion:

ROOT TEST. Suppose $\sum_{n=0}^{\infty} a_n$ is a series and $\lim_{n\to\infty} |a_n|^{1/n} = L$. Then:

1. If $L < 1$, then the series converges absolutely.
2. If $L > 1$, then the series does not converge.
3. If $L = 1$, the test tells us nothing, *i.e.* the series may converge or not.

3. Prove cases 1 and 2 of the Root Test. *[4]*

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can. You may work together, look things up, and use whatever tools you like, so long as you *write up your submission by yourself* and give due credit to your collaborators and any sources and tools you actually used.