Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Summer 2018
Practice Final Examination

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **A** and **B**, and, if you wish, part **C**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Any calculator; (all sides of) one aid sheet; one (1) brain (no neuron limit).

Part A. Do all four (4) of **1**–**4**.

1. Compute $\frac{dy}{dx}$ as best you can in any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. $y = \left(\frac{x+1}{x-1}\right)^2$ **b.** $y = \int_0^x te^{t^2}\,dt$ **c.** $\begin{array}{l} y = -\cos(t) \\ x = \sin(t) \end{array}$

d. $\ln(xy) = 0$ **e.** $y = \sin\left(\sqrt{x}\right)$ **f.** $y = x^\pi e^x$

2. Evaluate any *four* (4) of the integrals **a**–**f**. *[20 = 4 × 5 each]*

a. $\displaystyle\int \frac{e^{\sqrt{t}}}{2\sqrt{t}}\,dt$ **b.** $\displaystyle\int_0^{\pi/2} x\cos(x)\,dx$ **c.** $\displaystyle\int_0^1 \arctan(y)\,dy$

d. $\displaystyle\int_0^{\ln(2)} e^{-y}\,dy$ **e.** $\displaystyle\int_0^{\sqrt{\pi}} z\cos\left(z^2\right)\,dz$ **f.** $\displaystyle\int_0^{\pi/4} \tan^2(z)\,dz$

3. Do any *four* (4) of **a**–**f**. *[20 = 4 × 5 each]*

a. Let $f(x) = x^2 + 1$ and compute $f'(1)$ using the limit definition of the derivative.

b. Use the $\varepsilon - \delta$ definition of limits to verify that $\lim_{x \to 0} (2x - 1) = -1$.

c. Compute $\lim_{n \to \infty} \frac{n^2}{e^n}$.

d. Sketch the region between $y = x^2$ and $y = \sqrt{x}$, $0 \le x \le 1$, and find its area.

e. Find the equation of the tangent line to $y = \cos(x)$ at $x = \frac{\pi}{4}$.

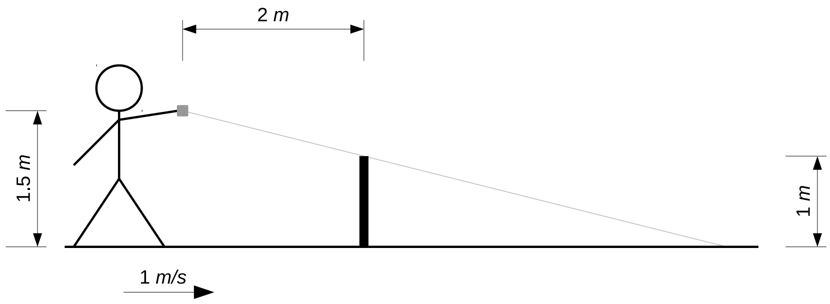
f. Find the number b such that $\displaystyle\int_0^b (2x+1)\,dx = 2$.

4. Find the domain and any and all intercepts, vertical and horizontal asymptotes, and maximum, minimum, and inflection points of $f(x) = e^{-x^2}$, and sketch its graph. *[12]*

Part B. Do any *two* (2) of **5–7**. *[28 = 2 × 14 each]*

5. What is the maximum area of a rectangle with its base on the x-axis and which has its two top corners on the semicircle $y = \sqrt{16 - x^2}$?

6. Meredith, carrying a lamp 1.5 m above the ground, walks at 1 m/s along level ground directly toward a 1 m tall post at night. How is the length of the shadow cast by the post in the lamplight changing at the instant that the lamp is 2 m from the post?



7. Sand is poured onto a level floor at the rate of 60 L/min. It forms a conical pile whose height is equal to the radius of the base. How fast is the height of the pile increasing when the pile is 2 m high? [The volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$.]

[Total = 100]

Part C. Bonus problems! If you feel like it and have the time, do one or both of these.

○. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots = \frac{\pi^2}{6}$. Assuming this is so [which it is], what is the series $\displaystyle\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = 1 + \frac{1}{9} + \frac{1}{25} + \cdots$ equal to? *[1]*

⊙. Write a haiku touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

Have some fun this summer,
and drop by next year to tell me about it!