

# Mathematics 1110H (Section A) – Calculus I: Limits, Derivatives, and Integrals

TRENT UNIVERSITY, Fall 2024

## Assignment #5

### Definite Integrals via the Right-Hand Rule

Due on Friday, 22 November.\*

A concept and a bit of notation first. The *definite integral* from  $a$  to  $b$  of  $f(x)$ , usually denoted by  $\int_a^b f(x) dx$ , is the weighted area between the graph of  $y = f(x)$  for  $x$  between  $a$  and  $b$  and the  $x$ -axis. “Weighted area” means that area above the  $x$ -axis is added and area below the  $x$ -axis is subtracted. Definite integrals are usually computed using antiderivatives, but they need to be defined in some other way first. There are also cases where we need to compute a definite integral, or at least approximate it, but the function in question has no antiderivative.

Before tackling this assignment, please read the accompanying handout, *Right-Hand Rule Riemann Sums*, which describes a still-somewhat-useful simplification of the definition of a definite integral. (If you have an interest in seeing a full-fat definition of the definite integral, which is beyond the scope of this course, check out the handout *A Precise Definition of the Definite Integral*.) If you haven’t already seen them in – or have forgotten them from – Assignment #3, please look up SageMath’s `sum` and `limit` commands before tackling this assignment. This assignment will also ask you to use SageMath’s `integral` command, which can be used to compute definite integrals as well as find “indefinite integrals” (*i.e.* generic antiderivatives).

Consider the definite integral  $\int_{-1}^3 (x^2 - 1) dx$ .

1. Sketch the region whose weighted area is computed by this definite integral. [1]
2. Set up the Right-Hand Rule formula – a limit of a sum as given on page 2 of the handout *Right-Hand Rule Riemann Sums* – for computing this definite integral. [2]
3. Evaluate the formula you obtained in solving question 2 using SageMath. [2]
4. Evaluate the formula you obtained in part b by hand. [4]

*Hint:* Some of the summation formulas you obtained in Assignment #3 are likely be useful here.

5. Evaluate the given definite integral using SageMath’s `integral` command. [1]

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\* Please submit your solutions, preferably as a single pdf, via Blackboard’s Assignments module. If that fails, please submit them to the instructor on paper or via email to [sbilaniuk@trentu.ca](mailto:sbilaniuk@trentu.ca) as soon as you can.