

MATH1110H-B-lab-2023-11-14-F01

November 21, 2023

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[1]: # MATH 1110H-B Lab F01 2023-11-14
#
# In this lab, we introduce the sum command, which allows us to
# compute finite and infinite sums. We need to declare a suitable
# variable, in this case n. Note that discrete (i.e. integer)
# tend to be chosen from the middle of the alphabet (more or less
# i to n, and sometimes p and q, especially for prime numbers).
# [We also introduce the integral command near the end.]
#
var('k')
#
# The following command adds up 1/k^2 for all k from 1 to 2:
#
sum(1/k^2,k,1,2)
```

[1]: 5/4

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[2]: # This one adds up 1/k^2 for all k from 1 to 100:
#
sum(1/k^2,k,1,100)
#
# # Note that SageMath delivers the answer as a giant fraction...
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[2]: 15895086941330378731122979285175538597023834985437098598894328348038181310903699
01/97218614443438103058965797667262314416197558399574624178272035470551798616524
8000

```
[3]: # ... though one could use the N command to get a decimal approximation.
#
N(sum(1/k^2,k,1,100))
```

[3]: 1.63498390018489

```
[4]: # One can also use the sum command to add up infinitely many terms:
#
sum(1/k^2,k,1,oo)
```

[4]: 1/6*pi^2

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[5]: # One can also define functions of a discrete variable, in this case
# using the sum command.
#
var('n')
s = function('s')(n)
s(n) = sum(1/k^2,k,1,n)
```

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[6]: # This can be used to get the same sum as above:
#
s(100)
```

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[6]: 15895086941330378731122979285175538597023834985437098598894328348038181310903699
01/97218614443438103058965797667262314416197558399574624178272035470551798616524
8000
```

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[7]: # In principle, one take take their limits too. The limit as n goes
# to infinity of the function defined above should be the same as the
# infinite sum above, but SageMath punts on the this computation.
#
limit(s(n),n=oo)
```

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[7]: limit(harmonic_number(n, 2), n, +Infinity)
```

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[8]: # It doesn't always punt in this way. With a different function...
#
t = function('t')(n)
t(n) = sum(1/2^k,k,1,n)
t(10)
```

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[8]: 1023/1024
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[9]: # ... Sagemath works out the limit correctly.
#
limit(t(n),n=oo)
```

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[9]: 1
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[10]: # The sum command has a similar syntax to the integral command:
#
integral(x^2-1,x,1,3)
```

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[10]: 20/3
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[11]: # And again, integrating the same function, but with over a larger
# interval#
#
integral(x^2-1,x,1,6)
```

[11]: 200/3

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