Implicit Differntiation

Solve for dy/dx = y when y isn’t necessarily a function of x.

Eg. X2+y2=1

Diagram 1

Differentiate both sides with respect to x.

d/dx X2+y2 = d/dx(1) = 0

d/dx X2+y2 = 2x + d/dx(y2) Derivative in respect to x

2x + [d/dx(y2)]\*dy/dx Chain Rule

2x + 2y dy/dx

Dy/dx=-x/y

Sine(xy) = ln(xy)

Dy/dx =?

d/dx sin(xy) = d/dx ln(xy)

d/dx ln(xy) = [d/d(xy) ln(xy)]\*d/dx(xy)

=1/xy ((dx/dx)y+x (dy/dx))

=1/xy (y+x (dy/dx))

=1/x + 1/y dy/dx

d/dx sin(xy) = [d/d(xy) sin(xy)] d/dx(xy)

=cos(xy) \* (y+x dy/dx)

cos(xy) \* (y+x dy/dx) = 1/xy (y+x (dy/dx))

=xy cos(xy) (y+x dy/dx) = y+x dy/dx

So either:

1. y+x dy/dx = 0

* dy/dx = -y/x

1. xycos(xy) = 1

This tells us nothing about dy/dx...

Dy/dx in case ii)

d/dx(xycos(xy)) = d/dx (1) = 0

=[d/dx (xy)] cos(xy) + (xy)[d/dx cos(xy)]

=[y+x dy/dx] cos(xy) + (xy)(-sin(xy))[y+x dy/dx]

=(y+ x dy/dx) cos(xy) = xysin(xy)(y+x dy/dx)

Either:

i)

or

1. cos(xy) = xysin(xy)

and it keeps happening...

Related Rates

Problem: A stone is dropped into a still pool of water creating a circular ripple that moves outward at a rate of 2m/s.

How is the are enclosed by the ripple changing when the ripple has radius 6m?

Diagram 2

Area = pi r2

= pi 62 = 36 pi m2

Want to know: dA/dt

We do know:

A= pi r 2

R=6

Dr/dt = 2m/s

What do we do?

dA/dt = d/dt (pi r2)

=pi (dr2/dr)(dr/dt)

=pi 2r dr/dt

When r = 6

dA/dt = 2pi \* 6 \* 2

= 24 pi m2/s

Lighthouse projects beam of light and it rotates. The beam rotates 4 times per minute. Beam moves along a straight shore. Lighthouse is 2.3km from the beach.

How quickly is the illuminated spot on the beach moving at this instant?

Distance from beach to lighthouse: 2.3

At the instance the light from normal: Fred(1.8)

Want to know:

Know

At the instant in question and only at that instant Fred is 1.8.

Chain Rule:

We need to know what is when

at the instant Fred = 1.8

So

Logarithmic Differentiation

(Chain Rule)

We want so we want to isolate it from the equation above.

To get we multiply by the reciprocal of which is.

Alternative method

Chapter 4

Maximum & Minimum Values

When does a function have a maximum (or minimum) value (on an interval)?

Possible candidates:

1. Points such that
2. Where is undefined.
3. End points of the interval

Example 1: Find the maximum and minimum values of on

* 0 is included – 3 is not

* Can never be equal to 0
* Undefined at x = 15 but 15 is not in the interval

So all we have to do is check the end points.

So the maximum on the interval is at 0 which is.

There is no minimum.

Example 2: What is the least length of fence needed to enclose a rectangular field of area 1000m2?