Chapter 2 Limits and Derivatives

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| 2.1 Tangent and Velocity Problems | Tangent – is derived from the Latin word tangens, which means touching. Thus a tangent to a curve is a line that touches the curve. In other words the tangent line should have the same direction as the curve at the point of contact. |
| 2.2 The Limit of a Function | “$\lim\_{x\to a}f\left(x\right)=L$” is read “The limit of f(x) as x approaches a is L”What can go wrong:* $f\left(a\right)$ might be undefined (a whole in the curve)
* No limit (point on a curve were curve is switches definition equation)
* Limit Exists outside of the rest of the curve

The precise definition of limits (Section 2.4) $\lim\_{x\to a}f\left(x\right)$ really means:For any $ε>0$, there is a $δ>0$, such that any x, if $0<\left|x-a\right|< δ$, then $\left|f\left(x\right)-L\right|<ε$.Computing Limits:1. $\lim\_{x\to a}C=C$
2. $\lim\_{x\to a}x=a$
3. $\lim\_{x\to a}x^{b}=a^{b}$
4. $\lim\_{x\to a}\left(f\left(x\right)+g\left(x\right)\right)= \lim\_{x\to a}f\left(x\right)+\lim\_{x\to a}g\left(x\right)$
5. $\lim\_{x\to a}Cf\left(x\right)=C\left(\lim\_{x\to a}f\left(x\right)\right)$
6. $\lim\_{x\to a}f\left(x\right)∙g\left(x\right)=\left(\lim\_{x\to a}f\left(x\right)\right)\left(\lim\_{x\to a}g\left(x\right)\right)$
7. $\lim\_{x\to a}\frac{f\left(x\right)}{g\left(x\right)}= \frac{\lim\_{x\to a}f\left(x\right)}{\lim\_{x\to a}g\left(x\right)}$

Ex: If the curve is continuous we can just stick in a for x:$\lim\_{x\to 2}\frac{x^{2}+3x-6}{\sqrt{x+2}}=\frac{4+6-6}{\sqrt{4}}=\frac{4}{2}=2$ Otherwise we must use the rules above:$\lim\_{x\to 2}\frac{x^{2}+3x-6}{\sqrt{x+2}}=\frac{\left(\lim\_{x\to 2}x^{2}\right)+\left(\lim\_{x\to 2}3x\right)-\left(\lim\_{x\to 2}6\right)}{\lim\_{x\to 2}\sqrt{x+2}}$ $=\frac{\left(\lim\_{x\to 2}x\right)^{2}+3\left(\lim\_{x\to 2}x\right)-6}{\sqrt{\left(\lim\_{x\to 2}x\right)+\left(\lim\_{x\to 2}2\right)}}=\frac{2^{2}+3\left(2\right)-6}{\sqrt{2+2}}=\frac{4}{2}=2$ Ex:$\lim\_{x\to 1}\frac{x^{3}-2x^{2}-x}{\left(x-1\right)^{2}}=\frac{0}{0}$ which is undefined as we cannot divide by 0.$\lim\_{x\to 1}\frac{x^{3}-2x^{2}-x}{\left(x-1\right)^{2}}=\lim\_{x\to 1}\frac{x\left(x^{2}-2x-1\right)}{\left(x-1\right)^{2}}=\lim\_{x\to 1}\frac{x\left(x-1\right)^{2}}{\left(x-1\right)^{2}}=\lim\_{x\to 1}x=1$  |