MATH 1101 2009 Midterm Test 2

February 10, 2010

Solution Name ____________________

1. (15 points) Evaluate the integral.

   (a) $\int x\cos 4x dx$

   Solution: Let $u = x$, $v' = \cos 4x$. $u' = 1$, $v = \frac{1}{4}\sin 4x$. Using integration by parts, we have

$$\begin{aligned} & \int x\cos 4x dx \\ = & \frac{x}{4}\sin 4x - \frac{1}{4}\int \sin 4x dx \\ = & \frac{x}{4}\sin 4x + \frac{1}{16}\cos 4x + C. \end{aligned}$$

$\square$

   (b) $\int \sin x\cos^5 x dx$

   Solution: Let $u = \cos x$, $du = -\sin x dx$. We have

$$\begin{aligned} & \int \sin x\cos^5 x dx \\ = & -\int u^5 du = -\frac{u^6}{6} + C = -\frac{\cos^6 x}{6} + C. \end{aligned}$$

$\square$

   (c) $\int \frac{x}{\sqrt{x^2-9}} dx$

   Solution: Let $u = x^2 - 9$. $du = 2x dx$. We have

$$\begin{aligned} & \int \frac{x}{\sqrt{x^2-9}} dx \\ = & \frac{1}{2}\int \frac{2x}{\sqrt{x^2-9}} dx = \frac{1}{2}\int \frac{1}{\sqrt{u}} du \\ = & \sqrt{u} + C = \sqrt{x^2-9} + C. \end{aligned}$$

$\square$

2. (2 points) Find the derivative of $g(x) = \int_x^{x^2} \frac{\sin t}{t} dt$.

   Solution: By Part 1 of the Fundamental Theorem of Calculus, we have

$$\begin{aligned} g(x) &= \int_x^1 \frac{\sin t}{t} dt + \int_1^{x^2} \frac{\sin t}{t} dt \\ &= -\int_1^x \frac{\sin t}{t} dt + \int_1^{x^2} \frac{\sin t}{t} dt \end{aligned}$$

$$g'(x) = -\frac{\sin x}{x} + \frac{\sin\left(x^2\right)}{x^2}(2x).$$

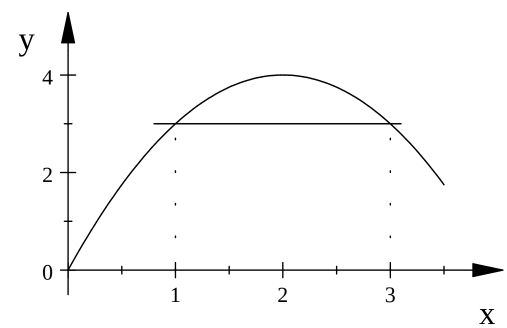
$\square$

3. (3 points) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 4x - x^2, y = 3$ about $x = -1$.

   Solution: Let $4x - x^2 = 3$. We have

$$\begin{aligned} x^2 - 4x + 3 &= 0 \\ (x-1)(x-3) &= 0. \end{aligned}$$

The intersections of these two curves have x-coordinates 1 and 3. At $x = 2$, $4x - x^2 = 8 - 4 = 4 > 3$. On the interval $[1, 3]$ the curve $y = 4x - x^2$ is above the line $y = 3$.



Since the rotation is about a vertical line, we use the shell method

$$\begin{aligned} V &= \int_1^3 2\pi\,(x-(-1))\left(4x - x^2 - 3\right) dx \\ &= \int_1^3 2\pi\,(x+1)\left(4x - x^2 - 3\right) dx. \end{aligned}$$

$\square$