MATH 1101Y 2009 Quiz 5 (a)

1. (1.5 pts) Find $\frac{dy}{dx}$ by implicit differentiation.

$$\sqrt{x^2+y^2} = \sin x \cdot \sin\left(y^2\right)$$

Solution: Differentiate both sides. We have

$$\begin{aligned}
\frac{1}{2\sqrt{x^2+y^2}}\left(2x+2y\frac{dy}{dx}\right) &= \cos x \cdot \sin\left(y^2\right) + \sin x \cdot \cos\left(y^2\right) 2y\frac{dy}{dx} \\
\frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\frac{dy}{dx} &= \cos x \cdot \sin\left(y^2\right) + 2y \sin x \cos\left(y^2\right)\frac{dy}{dx} \\
\left(\frac{y}{\sqrt{x^2+y^2}} - 2y\sin x\cos\left(y^2\right)\right)\frac{dy}{dx} &= \cos x \cdot \sin\left(y^2\right) - \frac{x}{\sqrt{x^2+y^2}}
\end{aligned}$$

$$\frac{dy}{dx} = \frac{\cos x \cdot \sin\left(y^2\right) - \frac{x}{\sqrt{x^2+y^2}}}{\frac{y}{\sqrt{x^2+y^2}} - 2y\sin x\cos\left(y^2\right)}.$$

□

2. (1.5 pts) Find $\frac{dy}{dx}$.

$$y = x^{\ln x}$$

Solution:

$$\ln y = \ln\left(x^{\ln x}\right) = (\ln x)(\ln x) = (\ln x)^2$$

$$\frac{1}{y}\frac{dy}{dx} = 2\ln x \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{2y}{x}\ln x = \frac{2x^{\ln x}}{x}\ln x.$$

□

3. (2 pts) Find $\frac{dy}{dx}$.

$$y = \sqrt[3]{\frac{x^3+4x^2-x+5}{3x^2+4x-1}}\sqrt{\frac{e^{5x}}{\sin x}}$$

Solution:

$$\begin{aligned}
\ln y &= \ln\left(\left(\frac{x^3+4x^2-x+5}{3x^2+4x-1}\right)^{\frac{1}{3}}\left(\frac{e^{5x}}{\sin x}\right)^{\frac{1}{2}}\right) \\
&= \ln\left(\frac{x^3+4x^2-x+5}{3x^2+4x-1}\right)^{\frac{1}{3}} + \ln\left(\frac{e^{5x}}{\sin x}\right)^{\frac{1}{2}} \\
&= \frac{1}{3}\left(\ln\left(x^3+4x^2-x+5\right) - \ln\left(3x^2+4x-1\right)\right) \\
&\quad + \frac{1}{2}\left(\ln\left(e^{5x}\right) - \ln(\sin x)\right)
\end{aligned}$$

$$
\begin{aligned}
\frac{1}{y}\frac{dy}{dx} &= \frac{1}{3}\left(\frac{3x^2+8x-1}{x^3+4x^2-x+5}-\frac{6x+4}{3x^2+4x-1}\right) \\
&\quad +\frac{1}{2}\left(5-\frac{\cos x}{\sin x}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dy}{dx} &= \sqrt[3]{\frac{x^3+4x^2-x+5}{3x^2+4x-1}}\sqrt{\frac{e^{5x}}{\sin x}}\cdot \\
&\quad \left[\frac{1}{3}\left(\frac{3x^2+8x-1}{x^3+4x^2-x+5}-\frac{6x+4}{3x^2+4x-1}\right)+\frac{1}{2}\left(5-\frac{\cos x}{\sin x}\right)\right].
\end{aligned}
$$

$\square$