Mathematics 110 – Calculus of one variable
TRENT UNIVERSITY, 2001-2002

Test #2
Friday, 8 February, 2002
Time: 50 minutes

1. Compute any *three* of the integrals **a-e**. *[12 = 3 × 4 ea.]*

a. $\displaystyle\int_{-\pi/2}^{\pi/2} \cos^3(x)\,dx$ **b.** $\displaystyle\int x^2\ln(x)\,dx$ **c.** $\displaystyle\int_0^1 (e^x)^2\,dx$

d. $\displaystyle\int \frac{e^{2x}\ln\left(e^{2x}+1\right)}{e^{2x}+1}\,dx$ **e.** $\displaystyle\int_1^e (\ln(x))^2\,dx$

2. Do any *two* of **a-c**. *[8 = 2 × 4 ea.]*

a. Compute $\int_0^1 (2x+3)\,dx$ using the Right-hand Rule.

b. Compute $\dfrac{dy}{dx}$ if $y = \int_0^{x^2} \sqrt{t}\,dt$ (where $x \geq 0$) without evaluating the integral.

c. Compute $\int_{-1}^{1} \sqrt{1-x^2}\,dx$ by interpreting it as an area.

3. Water is poured at a rate of 1 m^3/min into a conical tank (set up point down) 2 m high and with radius 1 m at the top. How quickly is the water rising in the tank at the instant that it is 1 m deep over the tip of the cone? *[8]*
(The volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.)

4. Consider the region in the first quadrant with upper boundary $y = x^2$ and lower boundary $y = x^3$, and also the solid obtained by rotating this region about the y-axis.

a. Sketch the region and find its area. *[4]*

b. Sketch the solid and find its volume. *[7]*

c. What is the average area of either a washer or a shell (your pick!) for the solid? *[1]*