Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2022
Alternate (Take-Home) Final Examination
06:00-18:00 (EDT) on Saturday, 23 April, via email.

Time: *3 hours.* *Brought to you by* Стефан Біланюк.

Instructions: Do parts **X**, **Y**, and P**Z**, and, if you wish, part **W**. Show all your work and justify all your answers. *If in doubt about something,* **ask!**

Aids: Open book, any calculator, but you may not look things up beyond what you can find on the MATH 1120H Blackboard site and the calculus archive page linked to there, nor give or receive any other help.

Part X. Do all four (4) of **1**–**4**.

1. Evaluate any *five* (5) of the integrals **a**–**f**. *[20 = 5 × 4 each]*

a. $\displaystyle\int_0^{\pi/4} \frac{\sec(x)\tan(x)}{\sqrt{1+\tan^2(x)}}\,dx$ **b.** $\displaystyle\int \frac{2}{\sqrt{4-z^2}}\,dz$ **c.** $\displaystyle\int_0^1 \frac{2+\arctan(y)}{1+y^2}\,dy$

d. $\displaystyle\int \sin^2(u)\cos^3(u)\,du$ **e.** $\displaystyle\int_0^\infty ve^{-v^2}\,dv$ **f.** $\displaystyle\int \frac{w}{\sqrt{4+w^2}}\,dw$

2. Determine whether the series converges in any *five* (5) of **a**–**f**. *[20 = 5 × 4 each]*

a. $\displaystyle\sum_{n=1}^\infty \frac{[\ln(n)]^2}{n}$ **b.** $\displaystyle\sum_{m=0}^\infty \frac{\sin(m)}{m^2+2m+1}$ **c.** $\displaystyle\sum_{i=1} \frac{(-1)^i}{i\sqrt{i+1}}$

d. $\displaystyle\sum_{j=1}^\infty \frac{2^j}{j^2}$ **e.** $\displaystyle\sum_{a=0}^\infty \frac{\sqrt{a}}{1+\sqrt{a}+a+a\sqrt{a}}$ **f.** $\displaystyle\sum_{k=1}^\infty \frac{(-1)^k(2k)!}{(2k+1)!}$

3. Do any *five* (5) of **a**–**f**. *[20 = 5 × 4 each]*

a. Find the Taylor series at 0 of $p(x) = (x+1)^3$ and find its radius of convergence.

b. Determine whether the series $\displaystyle\sum_{n=0}^\infty \frac{(-1)^n 3^{2n}}{e^n}$ diverges, converges conditionally, or converges absolutely.

c. Find the volume of the solid obtained by revolving the region below $y = 4$ and above $y = x^2$, where $0 \le x \le 2$, about the y-axis.

d. Use the Left-Hand Rule to compute $\displaystyle\int_0^1 (x+1)\,dx$.

e. Find the sum of the series $\displaystyle\sum_{k=1}^\infty \frac{(-2)^n}{3^{n+1}}$.

f. Find the area of the finite region between $y = 1$ and $y = x^4$.

4. Consider the region between $y = x^2$ and $y = 0$, where $0 \le x \le \pi$. Solid A is obtained by revolving this region about the line $x = 0$ and solid B is obtained by revolving the region about the line $y = 1$. Determine which of A and B has greater volume. *[12]*

Part Y. Do either *one* (1) of **5** and **6**. *[14]*

5. Consider the region between $y = \sqrt{1 - \frac{x^2}{9}}$ and $y = 0$, where $-3 \le x \le 3$.

- **a.** Find the area of this region. *[7]*
- **b.** Find the volume of the solid obtained by revolving the region about $y = 0$. *[7]*

6. Sketch the surface obtained by revolving the curve $y = \sin(x)$, for $\pi \le x \le 2\pi$, about the x-axis and find its area. *[14]*

Part Z. Do either *one* (1) of **7** or **8**. *[14]*

7. Find the Taylor series at 0 of $\arctan(x)$

- **a.** using Taylor's formula, *[9]* and
- **b.** without using Taylor's formula. *[5]*

8. Consider the power series $\sum_{n=0}^{\infty} (-1)^n (2n+1) x^{2n} = 1 - 3x^2 + 5x^4 - 7x^6 + \cdots$.

- **a.** Find the radius and interval of convergence of this power series. *[8]*
- **b.** Figure out what function has this power series as its Taylor series. *[6]*

[Total = 100]

Part W. Bonus problems! If you feel like it and have the time, do one or both of these.

9. Consider the following answers to a multiple-choice question:

a. The answer is *b*.
b. The answer is *c*.
c. The answer is *d*.
d. The answer is *e*.
e. None of the above.

Irrespective of the question, what should a student faced with this do? Explain! *[1]*

10. Write a haiku (or several :-) touching on calculus or mathematics in general. *[1]*

What is a haiku?

seventeen in three:
five and seven and five of
syllables in lines

ENJOY YOUR SUMMER!

P.S.: You can keep this question sheet. (Paper airplane, fire starter, the possibilities are endless! :-) The solutions to this exam will be posted to the course archive page at `http://euclid.trentu.ca/math/sb/1120H/` *in late April or early May.*