**Mathematics 1120H – Calculus II: Integrals and Series**
TRENT UNIVERSITY, Winter 2022

**Assignment #11**
**A Power Series For $\sqrt{1+x}$**
*Due on Friday, 8 April.*
*(May be submitted on paper or via Blackboard.*)*

Please show all your work. As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

If $r \in \mathbb{R}$ and $k \geq 1$ is an integer, then the *binomial of $r$ and $k$* is

$$\binom{r}{k} = \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!} \,.$$

Thus $\binom{r}{1} = r$, $\binom{r}{2} = \frac{r(r-1)}{2}$, $\binom{r}{3} = \frac{r(r-1)(r-2)}{6}$, and so on. To make various formulas work nicely, we let $\binom{r}{0} = 1$. Note that when $r$ is a positive integer, this coincides with the usual definition of binomial coefficients. *Newton's Binomial Theorem* extends the usual Binomial Theorem for expanding expressions like $(x+y)^n$ (for integer $n \geq 1$) and states that if $r$, $x$, and $a$ are real numbers with $|x| < |a|$, then

$$\begin{aligned}(a+x)^r &= \sum_{n=0}^{\infty} \binom{r}{n} a^{r-n} x^n \\ &= a^r + ra^{r-1}x + \frac{r(r-1)}{2}a^{r-2}x^2 + \frac{r(r-1)(r-2)}{6}a^{r-3}x^3 + \cdots \,.\end{aligned}$$

**1.** Suppose $|x| < 1$. We can expand $\sqrt{1+x} = (1+x)^{1/2}$ as a power series using Newton's Binomial Theorem. Find the radius and interval of convergence of this series. *[10]*

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`