Series VI

The Alternating Series Test (ch 11.4) and absolute convergence (ch 11.6)

Finally, finally we'll be able to handle the likes of

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$

<u>The Alternating Series Test</u>

Consider the series $\sum_{n=0}^{\infty} a_n$

Then if each $a_n \neq 0$ and

past some point
$$\begin{cases} (1) & a_{n+1} < 0 \iff a_n > 0 \quad \text{and} \\ (2) & |a_{n+1}| \leq |a_n| \quad \text{and} \\ (3) & \lim_{n\to\infty} |a_n| = 0 \end{cases}$$

if all these happen then the series converges

for $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ $\qquad a_n = \frac{(-1)^n}{n+1} \neq 0$ for all n

(1) if $a_{n+1} = \frac{(-1)^{n+1}}{n+2} < 0$ then n+1 is add so n is even, s $a_n = \frac{(-1)^n}{n+1} > 0$

[This reasoning is reversible so $a_{n+1} < 0 \iff a_n > 0$]

(2) $|a_{n+1}| = \frac{1}{n+2} < \frac{1}{n+1} = |a_n|$

(3) $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n+1} = 0$

$\therefore \sum_{n=}^{\infty} \frac{(-1)^n}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges by the Alt Series test

even though $\sum_{n=0}^{\infty} \frac{1}{n+1}$ diverges

<u>Def'n</u> A series $\sum_{n=0}^{\infty} a_n$ converges absolutely if $\sum_{n=}^{\infty} |a_n|$ converges

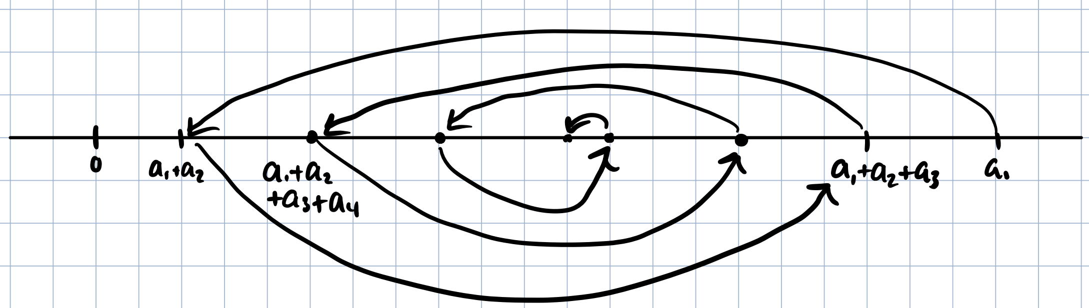
It converges conditanally if $\sum_{n=0}^{\infty} a_n$ converges but $\sum_{n=}^{\infty} |a_n|$ does not

Why does the AH. Series Test work?

Suppose $\sum_{n=0}^{\infty} a_n$ and it satisfies (1)-(3).

Since $\lim_{n\to\infty} |a_n| = 0$

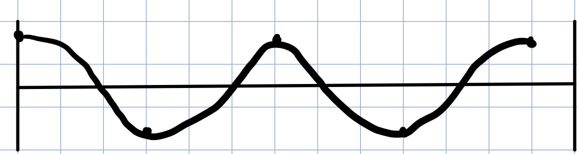
this homes in on some point



$a_1 > 0$
$a_2 < 0$
$a_3 > 0$
$a_4 < 0$

ex: $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln(n^2)}$ (1) This is an alternating series because $\ln(n^2) > 0$ if $n \geq 2$ and

$$\cos(n\pi) = \begin{cases} +1 \text{ if } n \text{ is even} \\ -1 \text{ if } n \text{ is odd} \end{cases}$$



(2) Also, $|a_{n+1}| = \left| \frac{\cos((n+1)\pi)}{\ln((n+1)^2)} \right| = \frac{1}{\ln((n+1)^2)}$

because $\ln((n+1)^2) > \ln(n^2)$

$|a_n| = \left| \frac{\cos(n\pi)}{\ln(n^2)} \right| = \frac{1}{\ln(n^2)}$

(3) $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \left| \frac{\cos(n\pi)}{\ln(n^2)} \right| = \lim_{n\to\infty} \frac{1}{\ln(n^2)} = 0$

Is this absolute or conditional convergent?

check if $\sum_{n=2}^{\infty} \left| \frac{\cos(n\pi)}{\ln(n^2)} \right| = \sum_{n=2}^{\infty} \frac{1}{\ln(n^2)}$ converges

$$\sum_{n=2}^{\infty} \frac{1}{\ln(n^2)} = \sum_{n=2}^{\infty} \frac{1}{2\ln(n)} = \frac{1}{2}\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$$

$\ln(n) < n$ for all $n \geq 1$

$$\Rightarrow \frac{1}{\ln(n)} > \frac{1}{n} \quad \text{— ,, —}$$

but $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges by the p-test since $p \leq 1$

so $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$ diverges by the Comparason test