Mathematics 1110H – Calculus I: Limits, derivatives, and Integrals
TRENT UNIVERSITY, Winter 2021

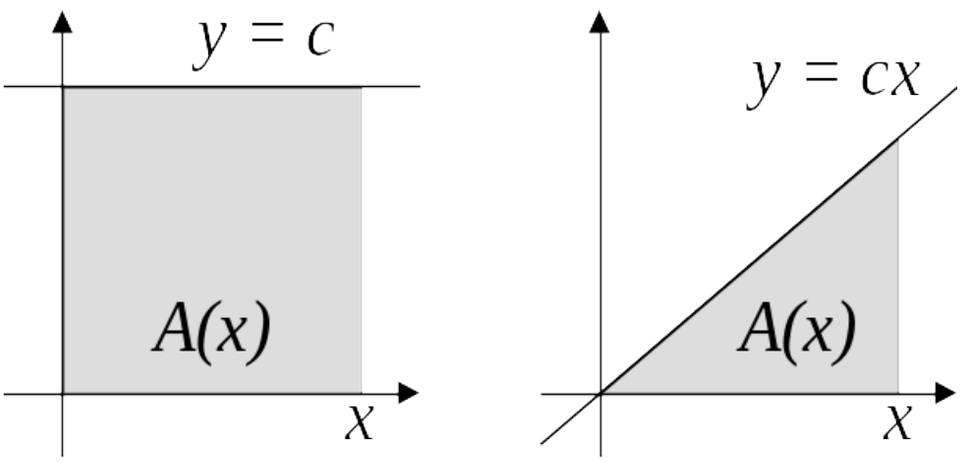
Assignment #5
Areas and Antiderivatives
Due on Friday, 26 March.

Submission: Scanned or photographed solutions are fine, so long as they are legible. Please try to make sure that they are oriented correctly – if they are sideways or upside down, they're rather harder to mark! Submission as a single pdf is strongly preferred, but other common formats are probably OK in a pinch. Also, please do not submit a file in one of `Maple`'s (or comparable program's) native format, though a printout of one to pdf would be more than acceptable. Please submit your solutions via Blackboard's Assignments module. If Blackboard does not acknowledge a successful upload, please try again. As a *last* resort, email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Restriction. Unlike most of our assignments, *you are not permitted to look things up for this assignment* (not that you should need to), but you may work together and ask the instructor for clarification and hints if you wish to do so.

The notation $\int_a^b f(x)\,dx$ represents the area between the graph of $y = f(x)$ and the x-axis for $a \leq x \leq b$. This notation is usually read as something like "the (definite) integral of $f(x)$ from a to b", by the way. We'll be seeing a lot of it fairly soon!

For this assignment we will not dip below the x-axis to avoid the twist that in $\int_a^b f(x)\,dx$ area below the x-axis counts negatively and is subtracted instead of added to the total area. Each of the two graphs below illustrates the area below the graph of a function from 0 to x, that is, in each case $A(x) = \int_0^x f(t)\,dt$ for the function $f(x)$ in question. (In each case, $c > 0$ is a constant.)



1. Use what you know of geometry to work out what $A(x)$ is as a function of x in each of the cases illustrated above. *[2]*

2. In each case given in the illustration, what is the relationship between $f(x)$ and $A(x)$? Aside from "$A(x)$ is the area under the graph of $f(x)$ from 0 to x", that is. *[2]*

3. Suppose $f(x) = cx^2$ for some constant $c > 0$ and $A(x) = \int_0^x f(t)\,dt$ gives the area under the graph of $f(x)$ from 0 to x. Based on the relationship between $f(x)$ and $A(x)$ you gave in answering question **2**, what should $A(x)$ be in terms of x? *[1]*

4. More generally, suppose $f(x) = cx^n$ for some constant $c > 0$ and integer $n \geq 1$, and $A(x) = \int_0^x f(t)\,dt$ gives the area under the graph of $f(x)$ from 0 to x. Based on the relationship between $f(x)$ and $A(x)$ you gave in answering question **2**, what should $A(x)$ be in terms of x? *[2]*

5. Even more generally, suppose $f(x)$ is any continuous function which is non-negative for $x \geq 0$. How could you try to work out the function $A(x)$ that give the area under the graph of $f(x)$ from 0 to x? Illustrate your ideas by working out the area under $y = \sin(x)$ for $0 \leq x \leq \pi$. *[3]*