Mathematics 1120H – Calculus II: Integrals and Series
TRENT UNIVERSITY, Winter 2020

Assignment #6
Gregory's Series
Due on Thursday, 2 April.
(Please submit your solutions using Blackboard's assignment module.)

The rate of convergence of the alternating series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$ got a little attention on Assignment #5 and its *Maple*-less alternative, Assignment #5.1. It was mentioned there that this series adds up to $\frac{\pi}{4}$. In this assignment we shall see why it does so, using a method similar to that used in the lecture notes for 2020-03-19 to show that the alternating harmonic series adds up to $\ln(2)$ if it is not rearranged.

1. Find a power series representation of $\arctan(x)$. *[6]*

Hint: Start with the fact that $\displaystyle\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$.

The power series representation obtained here is usually called *Gregory's series* after the Scottish mathematician and astronomer James Gregory (1638-1675) who rediscovered it in 1668. It had previously been discovered by the Indian mathematician and astronomer Madhava of Sangamagrama (1350-1410), and was independently rediscovered in 1676 by Gottfried Leibniz (1646-1716), one of the co-inventors of calculus.

2. Use the power series representation of $\arctan(x)$ obtained in **1** to conclude that $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots = \frac{\pi}{4}$. *[1]*

3. How practical is it to use Gregory's series to compute (approximations to) π? *[3]*